2ème partie :

Généralités

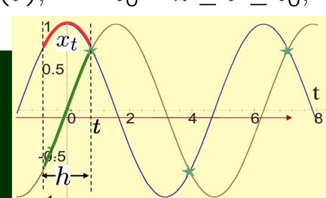
Jean-Pierre Richard, Ecole Centrale de Lille
Ecole d'Automne d'Automatique « La commande numérique des procédés industriels », Douz, Tunisie, 5-8 novembre 2006

Classification

- systèmes de type retardé

$$\dot{x}(t) = f(x_t, t, u_t)$$

$$\begin{aligned} x_t(\theta) &= x(t+\theta), & -h \le \theta \le 0, \\ u_t(\theta) &= u(t+\theta), & -h \le \theta \le 0, \\ x(\theta) &= \varphi(\theta), & t_0 - h \le \theta \le t_0, \end{aligned}$$



- systèmes de type neutre

$$\dot{x}(t) = f(x_t, \dot{x}_t, t, u_t)$$

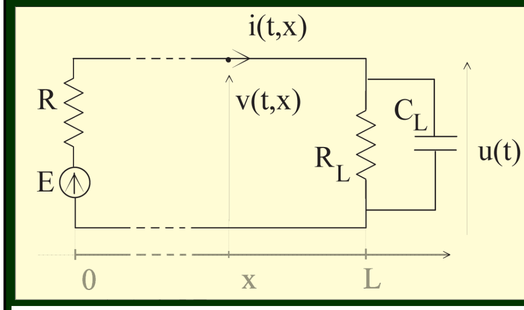
Classification

exemple 5

- système de type neutre :

$$\dot{x}(t) = f(x_t, \dot{x}_t, t, u_t)$$

ligne de transmission sans perte



$$\begin{cases} L\frac{\partial i}{\partial t} = -\frac{\partial v}{\partial x} \ , \\ C\frac{\partial v}{\partial t} = -\frac{\partial i}{\partial x} \ , \end{cases}$$

+ conditions aux limites (x=0, L) :

$$\begin{cases} v(t,0) = E - Ri(t,0) \ , \\ i(t,L) = \frac{1}{R_L}u(t) + C_L\frac{du}{dt}(t) \ . \end{cases}$$

transformation de d'Alembert $(v,i) \rightarrow (\phi,\psi)$:

$$v(t,x) = \phi(x-ct) + \psi(x+ct)$$
$$Zi(t,x) = \phi(x-ct) - \psi(x+ct)$$

$c = (LC)^{-1/2}, \quad Z = (L/C)^{1/2}$

$\tau = 2L/c, \ \kappa = (Z-R)/(Z+R)$

$$\dot{u}(t) - \kappa\dot{u}(t-\tau) + \alpha u(t) + \beta u(t-\tau) = \alpha E$$

Classification

exemple 6

- système de type retardé :

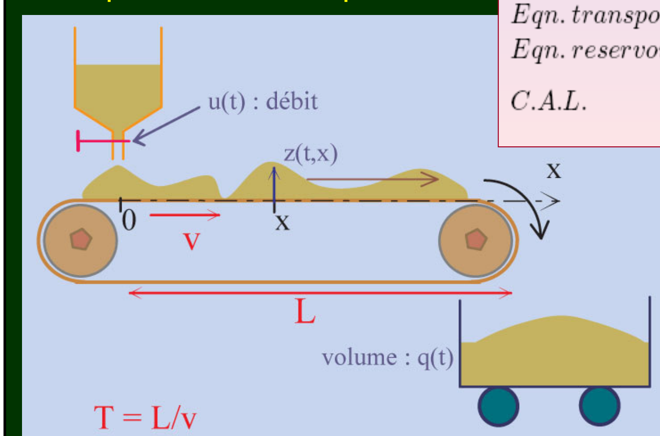
$$\dot{x}(t) = f(x_t, t, u_t)$$

équation de transport

$Eqn. transport :$ $\frac{\partial z}{\partial t} + v\frac{\partial z}{\partial x} = 0$

$Eqn. reservoir :$ $\dot{q}(t) = y(t)$

$C.A.L.$ $\begin{cases} y(t) = H\frac{\partial z}{\partial t}(t,L) \\ H\frac{\partial z}{\partial t}(t,0) = u(t) \end{cases}$



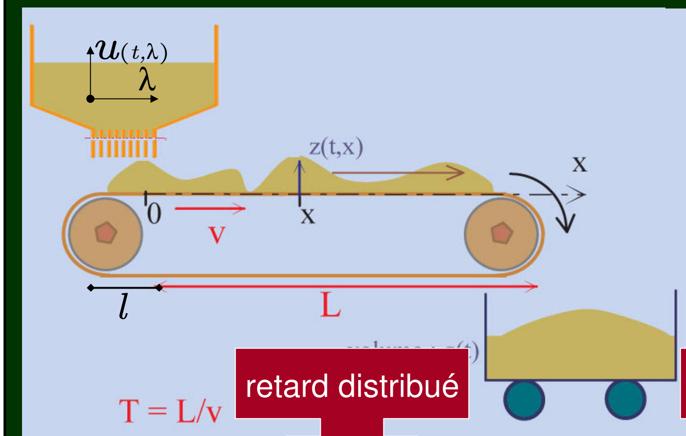
$$\dot{q}(t) = u(t - \frac{L}{v})$$

Classification

exemple 6 bis

- système de type retardé : $\dot{x}(t) = f(x_t, t, u_t)$

  équation de transport



retard distribué

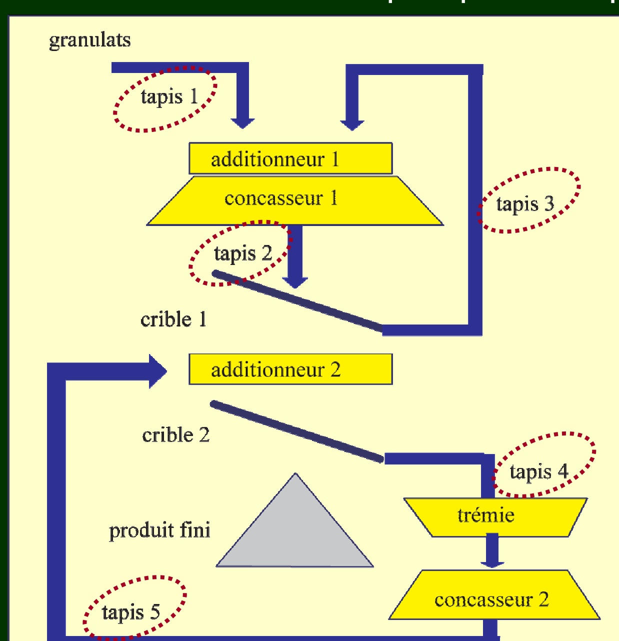
$$\dot{q}(t) = \int_{-l}^{0} u(t - \frac{L}{v} + \frac{\lambda}{v}, \lambda) d\lambda$$

retard ponctuel

$$\dot{q}(t) = u(t - \frac{L}{v})$$

Classification

NB : la réalité est un peu plus compliquée...



Classification

Exemple 7 : macro-modèle de réseau

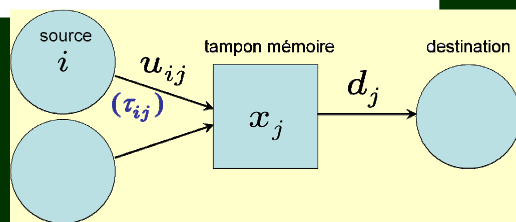
- modèle « par session » (ATM, TCP)

[Mascolo 99] → contrôle de congestion par prédicteur de Smith

$$\dot{x}_j(t) = \sum_{i=1}^{n_j} u_{ij}(t-\tau_{ij}) - d_j$$

x_j niveau du tampon mémoire associé à la session l_j
u_{ij} débit de la source i (commande)
n_j nombre de sessions partageant le tampon j
τ_{ij} retard de propagation de source i vers tampon j
d_j débit de service du tampon j (perturbation)

→ système de type retardé



Problème de Cauchy
(existence de solution pour un SàR)

- Notion de solution
- Condition de type Lipschitz
- Cas où le retard peut s'annuler

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Cauchy pb.

2.3 Notion of solution

System (S) : $\dot{x}(t) = f(t, x(t), x(t - \tau(t)))$

with $x(t) \in \mathbb{R}^n$, and $0 \leq \tau(t) \leq \tau$

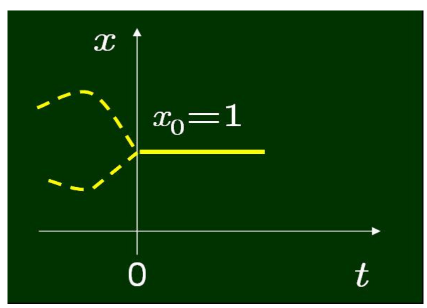
Let $\varphi : [-\tau, 0] \longrightarrow \mathbb{R}^n$ be an arbitrary map.

Definition: *A map $x(t) : [t_0 - \tau, t_0 + b) \longrightarrow \mathbb{R}^n$ s.t.*
1) $x(t_0 + s) = \varphi(s)$, for all s in $[-\tau, 0]$;
2) x is continuous over $[t_0, t_0 + b)$;
3) x satisfies (S) over $[t_0, t_0 + b)$ ($\dot{x}$ right-hand, Dini)
is called a solution of (S) with initial value φ at t_0.
If only one map satisfies these 3 points, then the solution is unique.

Remark: There is a weaker notion of solution, where
2)$\longrightarrow$ x *absolutely continuous* function over $[t_0, t_0 + b)$
3)$\longrightarrow$ x satisfies (S) *almost everywhere* on $[t_0, t_0 + b)$

Cauchy pb.

Remark



Even if unicity holds, different solutions may coincide after a finite time. For instance:

$$\dot{x}(t) = -x(t - \tau)[1 - x(t)],$$

$$x(t, \varphi) = 1 \quad (\forall t \geq 0)$$

for any $\varphi \in \mathcal{C}([-\tau, 0], \mathbb{R})$ such that $\varphi(0) = 1$.

(non-unicité de la réversion de trajectoire)

Cauchy pb.

2.4 Existence and uniqueness of solutions

For system (S) with $0 < \delta \leq \tau(t) \leq \tau_m$:

$$\dot{x}(t) = f(t, x(t), x(t - \tau(t))).$$

Consequence of the step method:
Given a continuous map $\varphi \in \mathcal{C}$, if the ODE

$$\dot{x}(t) = f_\varphi(t, x(t)) \equiv f(t, x(t), \varphi(t - \tau(t)))$$

has a (unique) solution, then there exists a (unique) solution of (S) with initial condition φ

From there, using classical Cauchy-Lipschitz cond^tions:
⟶Conditions of existence and uniqueness (I):
If f is a continuous map and satisfies a local Lipschitz condition in x,

$$\|f(t, x_2, y) - f(t, x_1, y)\| \leq K \|x_2 - x_1\|,$$

then for any initial condition $\varphi \in \mathcal{C}$, (S) has a unique solution, depending continuously on f and φ.

Cauchy pb.

If the delay can become zero $0 \leq \tau(t) \leq \tau_m$, the step method does not apply anymore

⇒ need of a general framework: FDEs [Myshkis 49]

$$(RFDE): \quad \dot{x}(t) = F_R(t, x_t) \qquad \text{(retarded type)}$$

Conditions of existence and uniqueness (II):
If F_R is a continuous map with local-Lipschitz cond. in its second (functional) argument, i.e.

$$\|F_R(t, \varphi_2) - F_R(t, \varphi_1)\| \leq K \|\varphi_2 - \varphi_1\|_{\mathcal{C}}, \ldots$$

then for any initial condition $\varphi \in \mathcal{C}([-\tau, 0], \mathbb{R}^n)$, (RFDE) has a unique solution, depending continuously on F_R and φ.

This condition is also necessary for previous system (S) (discrete, bounded, nonzero delay)

Formalismes

- FDEs (déjà présenté)
- Semi-groupes (opérateurs en dim. infinie)
- Géométrique (sur anneaux)
- Algébrique
- 2D (systèmes neutres)

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Formalismes

4.0 FDEs
(déja vu)

$$\dot{x}(t) = f\left(x_t, t, u_t\right), \qquad t \geq t_0$$

$$\begin{aligned} x_t(\theta) &= x(t+\theta), & -h \leq \theta \leq 0, \\ u_t(\theta) &= u(t+\theta), & -h \leq \theta \leq 0, \\ x(\theta) &= \varphi(\theta), & t_0 - h \leq \theta \leq t_0, \end{aligned}$$

→ très général (retards qcques, lin./non lin., ...)
→ commande par méthodes de type Lyapunov (en linéaire ou non lin. de type polytopique)

Formalismes

4.1 Operators in infinite dimension

Delfour72, Manitius78, Delfour-Karrakchou87, Bensoussan93...

$\mathcal{L}_2 = \mathcal{L}_2([-h, 0] \to \mathbb{R}^n)$ fncts with integrable square

$\mathcal{M}_2 = \mathbb{R}^n \times \mathcal{L}_2$ Hilbert space

$$\begin{aligned} \dot{x}(t) &= A_0 x(t) + A_1 x(t-h) + B_0 u(t), \\ y(t) &= C_0 x(t). \end{aligned}$$

$$\begin{cases} \dot{\overline{x}}(t) = \widetilde{A}\overline{x}(t) + \widetilde{B}u(t), \\ y(t) = \widetilde{C}\overline{x}(t) \end{cases}$$

$$\overline{x}(t) = [x(t), x_t] \in \mathcal{M}_2$$

$\widetilde{A}$ unbounded, closed, dense operator,

$\widetilde{B}$ and $\widetilde{C}$ bounded operators (if no input/output delay)

properties of semi-groups hold
general theory of infinite-dimensional, differential eqns

disadvantages:
1) abstract mathematics
2) yields controls in *distributed* form.

Formalismes

4.2 Geometric approach over polynomial rings

for linear systems with commensurate, constant delays

$\mathbb{R}[\nabla]$ commut. *ring* of polynom. in *delay operator* ∇
absence of inverse on $\mathbb{R}[\nabla]$ $\rightleftharpoons$ absence of advance ∇^{-1}

$$\begin{cases} \dot{x}(t) = \sum_{i=0}^{k} A_i x(t - i\delta) + \sum_{i=0}^{k} B_i u(t - i\delta) \\ y(t) = \sum_{i=1}^{k} C_i x(t - i\delta) \end{cases}$$

$$\downarrow$$

$$\begin{cases} \dot{x}(t) = \mathbf{A}(\nabla)x(t) + \mathbf{B}(\nabla)u(t), \\ y(t) = \mathbf{C}(\nabla)x(t), \end{cases}$$

$$\mathbf{A}(\nabla) \in \mathbb{R}^{n\times n}[\nabla], \mathbf{B}(\nabla) \in \mathbb{R}^{n\times m}[\nabla], \mathbf{C}(\nabla) \in \mathbb{R}^{p\times n}[\nabla]$$

$\Rightarrow x$ belongs to the *state-module* $\mathbb{R}[\nabla]^n$

Control: *polynomial* feedback $\Rightarrow$ polynomial system

$$\begin{aligned} u(t) &= -\mathbf{F}(\nabla)x(t) + v(t), \\ \mathbf{F}(\nabla) &\in \mathbb{R}[\nabla]^{m\times n} \end{aligned}$$

- *control* assumptions: *polynomial* control remains a limitation since realizability of concrete controllers needs:
  → *rational* fractions (precompensators, neutral syst)
  → *distributed* delays (finite spectrum assignment)

Formalismes

4.3 Systems over rational rings

ring $R[\nabla] \longrightarrow$ *field* $R(\nabla)$ *of rational fractions in* ∇
$\longrightarrow$ *allows dynamic feedback, but realizability?*
$\mathcal{R}_u(\nabla)$ = subring of the irreducible, rational fractions in ∇, which denominator has a non-zero constant term:

$$\mathcal{R}_u(\nabla) = \left\{ \frac{p(\nabla)}{q(\nabla)} \in \mathbb{R}(\nabla),\ q(0) \neq 0 \right\}.$$

Examples:

$\frac{1}{1+\nabla}$belongs to $\mathcal{R}_u(\nabla)$ $\qquad y(t) = -y(t-\delta) + u(t)$

$\frac{1}{\nabla}$does not $\qquad y(t) = u(t+\delta)$ is anticipative

Theorem (causality) [Picard, Lafay, Kucera 96]

$$\frac{p(s,\nabla)}{q(s,\nabla)} = \frac{p_0(\nabla) + ... + s^r p_r(\nabla)}{q_0(\nabla) + ... + s^k q_k(\nabla)}, \quad r \leq k$$

is causal $\iff q_k(\nabla) \in \mathcal{R}_u(\nabla)$ (ring property).

Advantage *of models over* $\mathbb{R}_u(\nabla)$*:*
dynamic feedback law defined over $\mathcal{R}_u(\nabla)$
$\Rightarrow$ resulting system remains in the same class

Formalismes

4.4 Algebraic formalism for distrib. delays

Manitius-Olbrot 79, Kamen-Khargonekar-Tannenbaum 85, Watanabe 96

Example:

$$u \longrightarrow y, \qquad y(t) = \int_{h_1}^{h_2} f(\theta)u(t-\theta)d\theta,$$

$$\frac{Y(s)}{U(s)} = F(s) = \int_{h_1}^{h_2} f(\theta)e^{-s\theta}d\theta$$

$ex \longrightarrow$ zero-holder operator $\frac{1-e^{-sh}}{s} = \int_0^h e^{-s\theta}d\theta.$

$\longrightarrow$ *explicit consideration of the relation* $s \rightleftharpoons \nabla \equiv e^{-s\delta}$

$$\mathcal{G} = \left\{ \mathcal{L}(\text{realizable, distributed delay}) \in \mathbb{R}(s, e^{-s\delta}) \right\}$$

$\longrightarrow \mathcal{E}$ = ring of *pseudo-polynomials* (analytic fct)

$$\mathcal{E} = \mathbb{R}[e^{-s\delta}] \cup \mathcal{G} \qquad \text{[Brethé, Loiseau]}$$

$\mathcal{E}$ is isomorphic to the quasi-polynomials ring $\mathbb{R}[s, e^{-s\delta}]$
$\mathcal{E}$ is a domain of Bezout ($\longrightarrow$ finite-spectrum assig$^{\text{mt}}$)

Formalismes

4.5 Generalization to nonlinear systems

[Moog, Marquez-Martinez 00]

Non commutative polynomial ring $\mathcal{K}[\nabla]$,
$\mathcal{K}$ field of meromorphic (ratio of analytic) functions:

$$\begin{aligned} \dot{x}_1(t) &= x_1(t)x_2(t-1) \equiv x_{1,0}x_{2,1} = x_{1,0}\delta x_{2,0} \\ \dot{x}_2(t) &= u_1(t) + x_2(t-2) \equiv u_{1,0} + x_{2,2} \end{aligned}$$

non commutative :

$$\begin{aligned} x_{1,0}\delta x_{2,0} &= x_{1,0}x_{2,1} \\ &\neq \\ \delta x_{1,0}x_{2,0} &= x_{1,1}x_{2,0} \end{aligned}$$

$\mathcal{K}[\nabla]$ integer ring with Euclidean left-division.

Formalismes

4.6 2-D models and neutral systems

Roesser models [1975]:
s = derivation operator ω = h-advance operator.

$$\begin{cases} sX = A_0X + A_2Z + B_0U, \\ \omega Z = A_3X + DZ + B_3U, \end{cases}$$
$$Y = C_1X + C_2Z.$$

Example:

$$A_2 = I,\ A_3 = A_1 + DA_0,\ B_3 = B_1 + DB_0,$$
$$\Downarrow$$
$$\dot{x}(t)\text{-}D\,\dot{x}(t\text{-}h) = A_0x(t)+A_1x(t\text{-}h)+B_0u(t)+B_1u(t\text{-}h)$$

$\Rightarrow$ allows connection with previous results
$\longrightarrow$ realization [Eising 78]
$\longrightarrow$ stabilization [Zak 86]
$\longrightarrow$ factorization & model matching [Loiseau, Brethé].
$\longrightarrow$ equivalence with the question of realization over $\mathcal{R}_u(\nabla)$ [Picard].

Commande

- Conditions structurelles (existence)
- Limite des méthodes « dim. finie »
- Prédicteur de Smith, le principe
- Approche Lyapunov
- Approche algébrique
- Approche fonctionnelle

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Commande

exemple

$$\dot{x}(t) = x(t) + u(t-1)$$

n'est pas stabilisable avec $u(t) = -kx(t)$

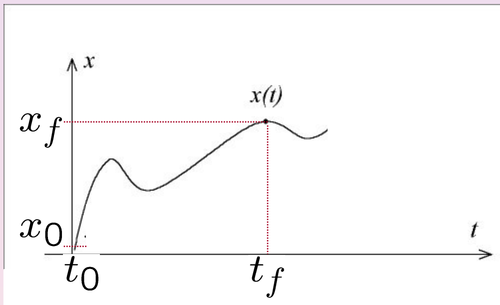
Structure de la commande ?

$\Rightarrow$ Besoin de connaître les propriétés structurelles du système.

Commande

Exemple de propriété structurelle : COMMANDABILITE

$\exists$? commande u faisant passer l'***état*** $x(t_0)=x_0$ en $x(t_f)=x_f$?



1) controllability: to reach a *function* instead of a point.

2) control delay $\Rightarrow$ *minimum reaching time*.

$\Rightarrow$ the notion of reachable sets ("orders" of Kalman-like controllability chains) has to be completed by associating a "class" depending on the time needed to achieve the control.

3) realization of the control law: $u(t) = g(x_t)$

$\rightarrow$ one may prefer "memoryless" controls $u(t) = g(x(t))$ or point-wise-delayed $u(t) = g(x(t), x(t-h_i))$.

$\Rightarrow$Several notions (controllability$\downarrow$)

Syst. dim. finie : état $x(t_0)=x_0$ à $x(t_f)=x_f$

En linéaire-stationnaire (principe de superposition OK) :

- si $\exists$ u, indépendant de t_f-t_0 et : ce qui est possible en T l'est en T/100 (!)
- si passage possible de x_0 à x_f, alors possible de x_f à x_0

Ce n'est plus vrai non linéaire

Syst. dim. infinie : état = fonction x_{t_f}

Ce n'est plus vrai non plus, même en linéaire

Commande

	controlability	abridged def	restrict
funct-ional	1 $\mathcal{M}_2$-strict	$\exists u, \exists t_1,$ $x_{t_1}=\varphi_1$	-
	2 $\mathcal{M}_2$-approx.	$\exists u_n, \exists t_1,$ $\lim[x_{t_1}]_{u_n}=\varphi_1$	-
	3 absolute	$\exists u, u_{t_1}=0,$ $x_{t_1}=\varphi_1$	lin, ∇u
	4 $(\psi, \mathbb{R}^n)$-fct	$\exists u, \exists t_1,$ $x_{t_1}=\psi \in \mathcal{C}$	lin, ∇x
	5 spectral	$\exists u,$ $\sigma_{A(e^{-\delta s})}=\{\lambda_i\}$	lin, coms.
point-wise	6 $\mathbb{R}^n$-	$\exists u, \exists t_1,$ $x(t_1)=x_1$	lin, coms.
	7 strong $\mathbb{R}^n$-	$\exists u, \forall t_1,$ $x(t_1)=x_1$	lin, coms.
	8 $\mathbb{R}^n$-to 0	$\exists u, \exists t_1,$ $x(t_1)=0$	lin, coms.
$u(t)$ type	9 (strong) over $\mathbb{R}[\nabla]$	polyn. $u(t)$ $=K(\nabla)x(t)$	lin, coms.
	10 (weak) over $\mathbb{R}(\nabla)$	rational $L(\nabla)u(t)$ $=K(\nabla)x(t)$	lin, coms.

for LTI with coms delays: $\boxed{9} \Rightarrow \boxed{3} \Rightarrow$ / $\boxed{2} \Rightarrow \boxed{5} \Rightarrow$ $\boxed{10} \Rightarrow \boxed{6}$

Commande

OK, c'est de l'existence...
mais **comment calculer cette commande ?**

$$(\Sigma) \quad \begin{cases} \dot{x}(t) = \sum_{i=0}^{q} A_i x(t-ih) + \sum_{i=0}^{q} B_i u(t-ih) \\ y(t) = \sum_{i=0}^{q} C_i x(t-ih) \end{cases}$$

Trouver un retour de sortie (dynamique) ou d'état $u(t)$ t.q. le système bouclé soit asympt. stable

2 méthodes :

- Approximation en dim. finie
  $\exp(-hs) \to \frac{N(s)}{D(s)}$ ex. approximants de Padé
  $$\frac{1-\frac{h}{2}s}{1+\frac{h}{2}s} = 1 - hs + \frac{h^2}{2}s^2 - \frac{h^3}{4}s^3 + O(s^4)$$
- conserver le système de dim. infinie

Commande

Première approche :

Problème :

$\Rightarrow$ syst. de grande dimension $\Rightarrow$ contrôleurs complexes

Ordre de l'approximation, stabilité numérique des algorithmes ?

Seconde approche :

- Besoin d'outils spécifiques
- Régulateurs de dimension infinie
  $\Rightarrow$ Implantation / approximation ?

... cf. la suite au cours suivant (Michel Dambrine)

Petite bibliographie

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