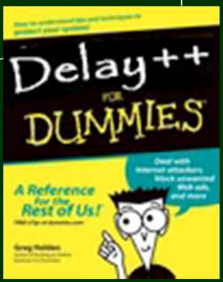
A delay, what does it change ?

Time-delay systems are also called systems with after effect or dead-time, hereditary systems, equations with deviating argument or differential-difference equations. They belong to the class of functional differential equations which are infinite dimensional, as opposed to ordinary differential equations. In spite of their complexity, they however often appear as simple infinite-dimensional models in the very complex area of partial differential equations. After the presentation of some motivating examples, the talk will try to show main differences arising from the presence of deviating time-arguments in the dynamics, seen from different points of view : state, solutions, stability, identification...

A delay, what does it change?



The point of view of a control guy…

Jean-Pierre Richard

LAGIS – CNRS
&
ALIEN – INRIA

GDR 2984 DYCOEC – DYnamique et COntrôle des Ensembles Complexes
WORKSHOP ON « SYNCHRONIZATION, CONTROL & BIO-DYNAMICS »
FEMTO ST – Besançon – 8 November 2010

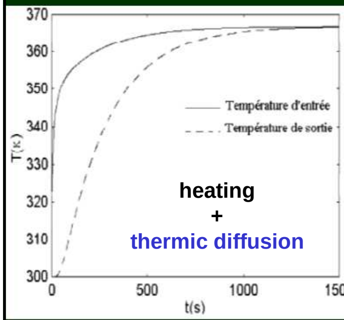
http://syner.free.fr
jean-pierre.richard@ec-lille.fr

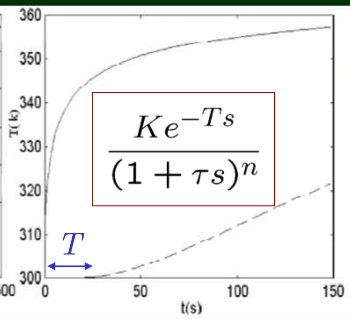
Delays : Classical examples

Strejć/Broïda – like models for industrial control

- frequent in process engineering
- simple and generic approximation
- PID controler ? ... poor results if $T > \tau$

➔ Smith predictor or « GPID » ...

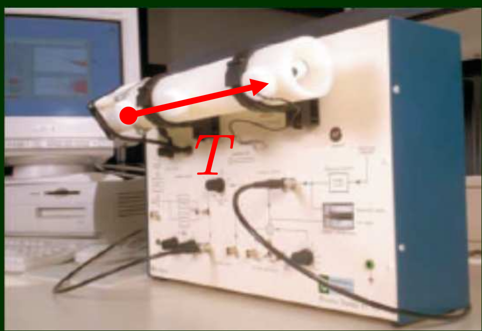


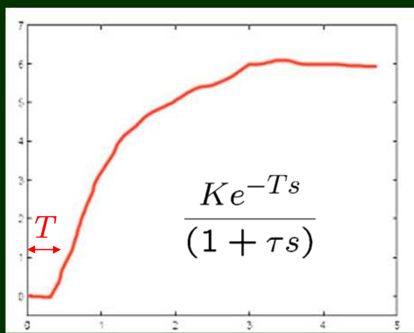


Ex: Steam Genererator LAGIS

Delays : Another classical example

... a great standard of control classes (Feedback)





… and of systems with transport phenomena

$$\left.\begin{array}{l} T \approx 20.10^{-3} sec, \\ \tau \approx 1 \;\; sec. \end{array}\right\} \text{PID ok}$$

Top 25 Hottest Articles

Decision Sciences > Automatica
July to September 2008

RSS Blog This! Print **Show condensed**

automatica

1. **Robust output-feedback model predictive control for systems with unstructured uncertainty**
   Automatica, Volume 44, Issue 8, August 2008, Pages 1933-1943
   Lovaas, C.; Seron, M.M.; Goodwin, G.C.
   Cited by SciVerse Scopus (13)
2. **Linear systems theory revisited**
   Automatica, Volume 44, Issue 7, July 2008, Pages 1686-1696
   Van Willigenburg, L.G.; De Koning, W.L.
   Cited by SciVerse Scopus (1)
3. **New nonlinear residual feedback observer for fault diagnosis in nonlinear systems**
   Automatica, Volume 44, Issue 9, September 2008, Pages 2222-2229
   Narasimhan, S.; Vachhani, P.; Rengaswamy, R.
   Cited by SciVerse Scopus (4)
4. **Time-delay systems: an overview of some recent advances and open problems**
   Automatica, Volume 39, Issue 10, October 2003, Pages 1667-1694
   Richard, J.-P.
   Cited by SciVerse Scopus (575)
5. **A new delay system approach to network-based control**
   Automatica, Volume 44, Issue 1, January 2008, Pages 39-52
   Gao, H.; Chen, T.; Lam, J.
   Cited by SciVerse Scopus (141)
6. **Delay-dependent robust stability and @?"2-gain analysis of a class of nonlinear time-delay systems**
   Automatica, Volume 44, Issue 8, August 2008, Pages 2006-2018
   Coutinho, D.F.; de Souza, C.E.
   Cited by SciVerse Scopus (3)
7. **Global output-feedback stabilization for a class of stochastic non-minimum-phase nonlinear systems**
   Automatica, Volume 44, Issue 8, August 2008, Pages 1944-1957
   Liu, S.J.; Jiang, Z.P.; Zhang, J.F.
   Cited by SciVerse Scopus (9)
8. **Average consensus problems in networks of agents with delayed communications**
   Automatica, Volume 44, Issue 8, August 2008, Pages 1985-1995
   Bliman, P.A.; Ferrari-Trecate, G.
   Cited by SciVerse Scopus (23)
9. **On input-to-state stability of systems with time-delay: A matrix inequalities approach** - Short communication
   Automatica, Volume 44, Issue 9, September 2008, Pages 2364-2369
   Fridman, E.; Dambrine, M.; Yeganefar, N.
   Cited by SciVerse Scopus (8)

1) an applied topic,
2) an already old topic
[Smith 1959],

but, also...

a contemporary *research* topic

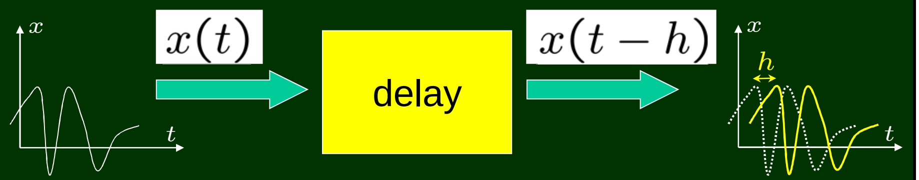
5 among the hottest 9

ELSEVIER AUTOMATICA Top 10 Cited (articles published in the last five years)
Extracted from **Scopus** (on *Sun Oct 31 20:08:05 GMT 2010*)

- **Network-based robust H∞ control of systems with uncertainty** — 325
  Volume 41, Issue 6, 2005, Pp 999-1007
  Yue, D. | Han, Q.-L. | Lam, J.
- **Delay-range-dependent stability for systems with time-varying delay** — 212
  Volume 43, Issue 2, 2007, Pp 371-376
  He, Y. | Wang, Q.-G. | Lin, C. | Wu, M.
- **Delay-dependent stabilization of linear systems with time-varying state and input delays** — 181
  Volume 41, Issue 8, 2005, Pp 1405-1412
  Zhang, X.-M. | Wu, M. | She, J.-H. | He, Y.
- **Stabilization of linear systems over networks with bounded packet loss** — 156
  Volume 43, Issue 1, 2007, Pp 80-87
  Xiong, J. | Lam, J.
- **Stability and L2-gain analysis for switched delay systems: A delay-dependent method** — 149
  Volume 42, Issue 10, 2006, Pp 1769-1774
  Sun, X.-M. | Zhao, J. | Hill, D.J.
- **Bilateral teleoperation: An historical survey** — 148
  Volume 42, Issue 12, 2006, Pp 2035-2057
  Hokayem, P.F. | Spong, M.W.
- **A new delay system approach to network-based control** — 139
  Volume 44, Issue 1, 2008, Pp 39-52
  Gao, H. | Chen, T. | Lam, J.
- **Absolute stability of time-delay systems with sector-bounded nonlinearity** — 132
  Volume 41, Issue 12, 2005, Pp 2171-2176
  Han, Q.-L.
- **Robust integral sliding mode control for uncertain stochastic systems with time-varying delay** — 112
  Volume 41, Issue 5, 2005, Pp 873-880
  Niu, Y. | Ho, D.W.C. | Lam, J.

9 among the
last 5 years top 10 !

Much a do about delay ?

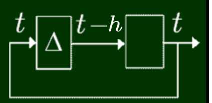


- applied problem

  engineering (NCS, telecom, crypto, real time), biology, populations, nuclear...

- still open in many cases

  closed-loop, variable delays, unknown delays, identification...



- the « simplest » infinite dimension problem

  functional equations, particular case of PDEs

- surprising properties

  damaging/improving by adding delays, data-sampling model...

Contents

Distinctive features of TDS?

Illustrative examples

- 1[rst] example (remote crtl.) → basic notions (stability, state, inf. dim.)
- 2[nd] example : variable delay → counter-example
- 3[rd] example : sampling → delay for modelling
- 4[th] example : **N**etworked **C**ontrol **S**ystem (master-slave)
- goodies…

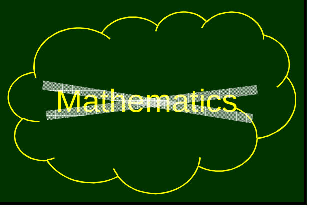
Cauchy's problem for TDS

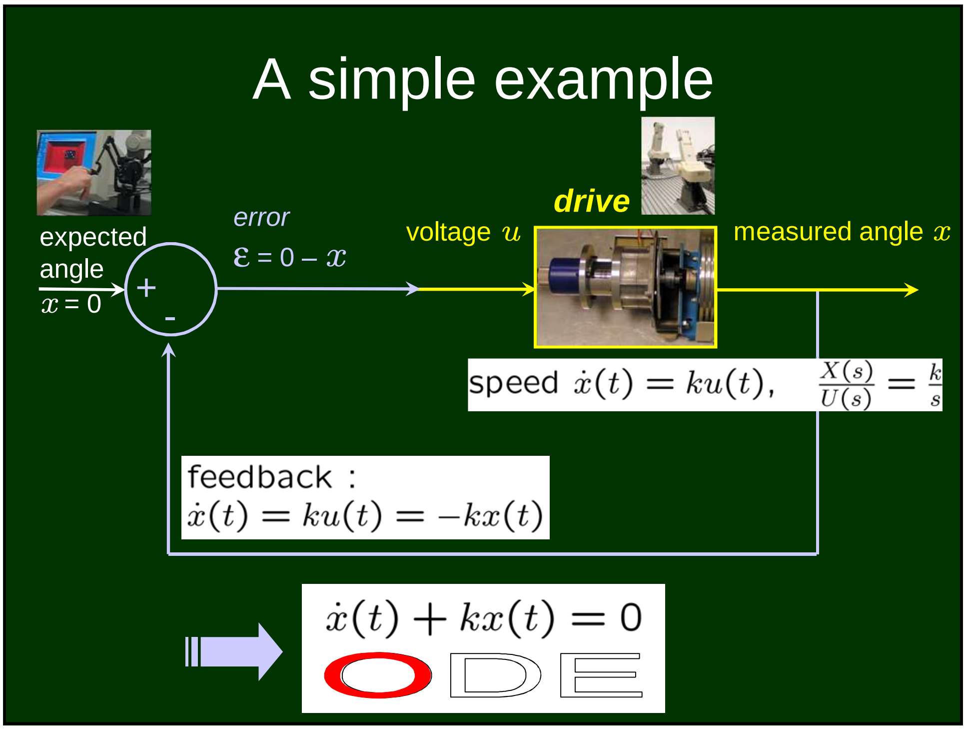
- Notion of solution
- Lipschitz-type condition
- And if the delay can vanish…

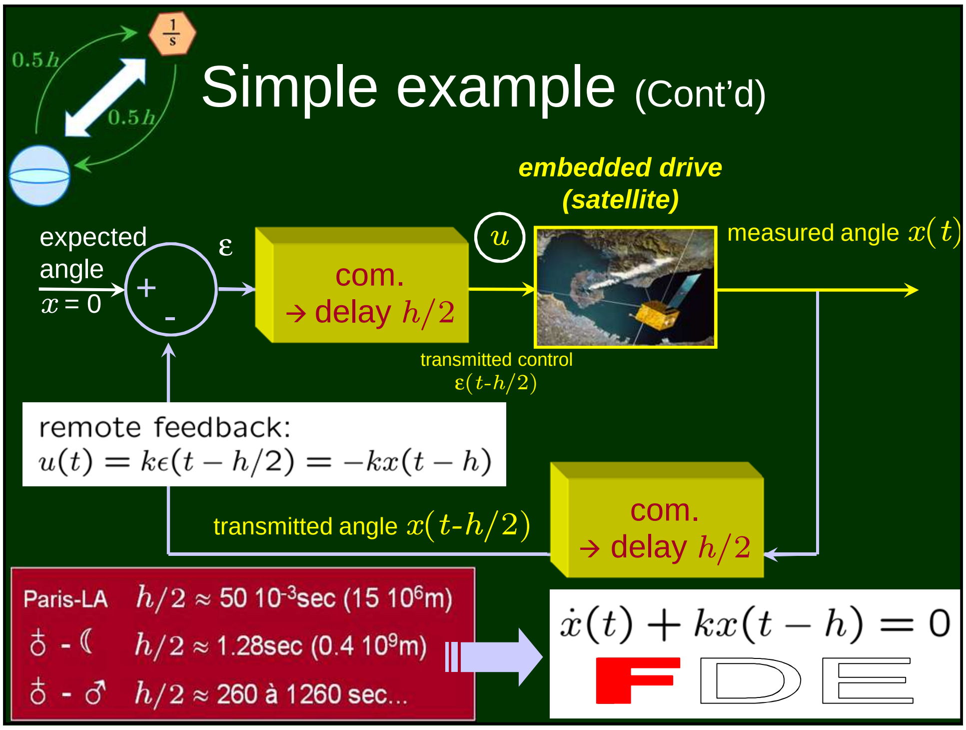
Stability and Lyapunov

- the LTI case
- 1[rst] Lyapunov (small states)
- small delays
- 2[nd] Lyapunov

Some words about identification




A simple example
expected angle
$x = 0$
+
-
error
$\varepsilon = 0 - x$
voltage u
drive
measured angle x
speed $\dot{x}(t) = ku(t)$, $\frac{X(s)}{U(s)} = \frac{k}{s}$
feedback :
$\dot{x}(t) = ku(t) = -kx(t)$
$\dot{x}(t) + kx(t) = 0$
ODE


Simple example (Cont'd)
$0.5h$
$0.5h$
$\frac{1}{s}$
embedded drive
(satellite)
expected angle
$x = 0$
+
-
ε
com.
→ delay $h/2$
u
measured angle $x(t)$
transmitted control
$\varepsilon(t-h/2)$
remote feedback:
$u(t) = k\epsilon(t - h/2) = -kx(t - h)$
transmitted angle $x(t-h/2)$
com.
→ delay $h/2$
Paris-LA $h/2 \approx$ 50 10^{-3}sec (15 10^6m)
♁ - ☾ $h/2 \approx$ 1.28sec (0.4 10^9m)
♁ - ♂ $h/2 \approx$ 260 à 1260 sec...
$\dot{x}(t) + kx(t - h) = 0$
FDE

Simple example (cont'd)

$\dot{x}(t) + x(t-h) = 0$ (case $h=1,\ k=1$)

$\dot{x}(t) = -x(t-1)$

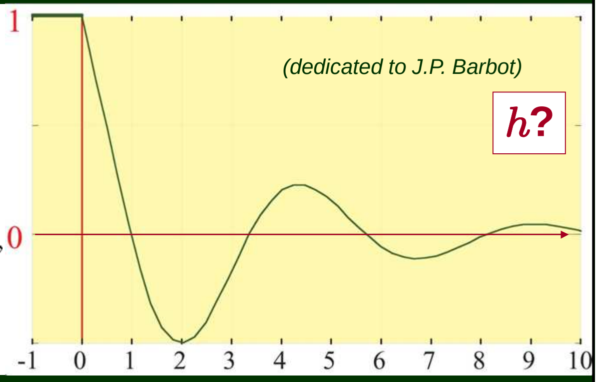
I.C. $t=0$: $x(t=0)=1$??

$t \in [-1,0]$: $x(t) = 1$ (I.C.)

$t \in [0,1]$: $x(t) = 1 - t$,

$t \in [1,2]$: $x(t) = \frac{1}{2} - t + \frac{t^2}{2}$,

etc.



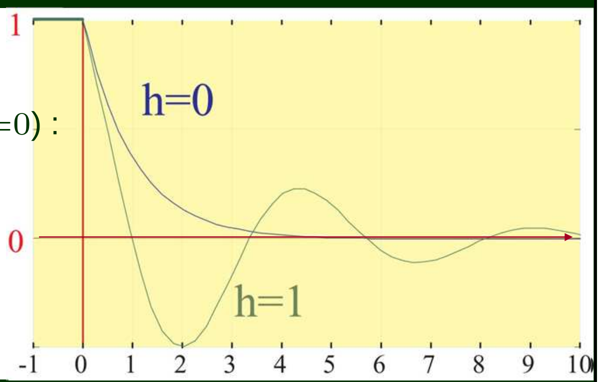
Simple example (cont'd)

$\dot{x}(t) + x(t-h) = 0$ w.r.t. h ?

$\dot{x}(t) + x(t-1) = 0$

to be compared with (h=0) :

$\dot{x}(t) + x(t) = 0$

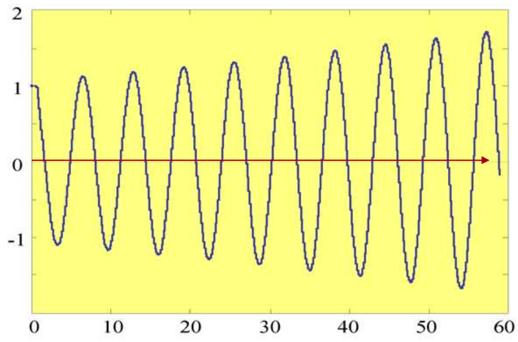


Simple example (cont'd)

$$\dot{x}(t) + x(t-h) = 0 \qquad h = 1.6$$

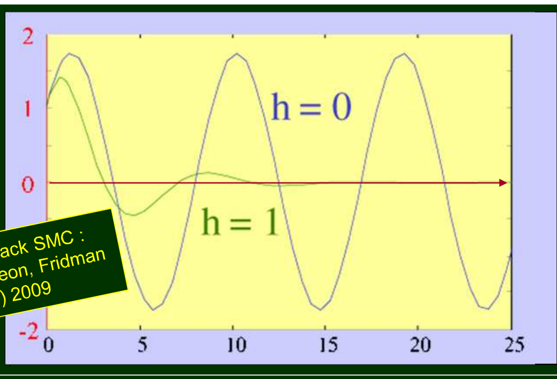
$$\dot{x}(t) + x(t-1.6) = 0$$

$t \in [-1, 0]: \; x(t) = 1$ (same I.C.)



(intermezzo...)

$$\ddot{y}(t) + y(t) - \tfrac{1}{2} y(t-h) = 0$$



see also output feedback SMC :
Seuret, Edward, Spurgeon, Fridman
IEEE TAC 54(2) 2009

a delay can have a stabilizing effect as well

here, derivative effect: $y(t-h) \approx y(t) - h\dot{y}(t)$

Back to the simple example...

$\dot{x}(t) + x(t-h) = 0$ $\quad h = \frac{\pi}{2}$

$\dot{x}(t) + x(t - \frac{\pi}{2}) = 0$

$\frac{d}{dt}x(t) = -x(t - \frac{\pi}{2}) = 0$

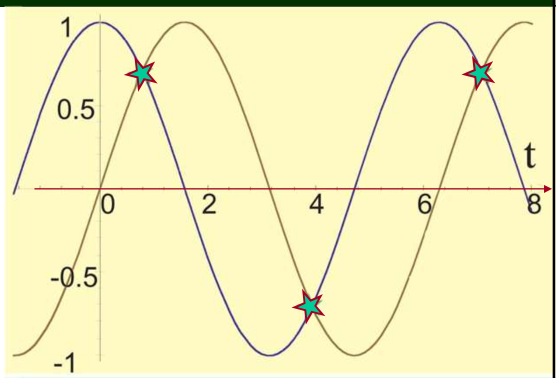
$x(t) = \cos\ t,$

$x(t) = \sin\ t,$

...

→ « state » notion ?

some variable $\phi(t)$ generating a unique solution starting at time t

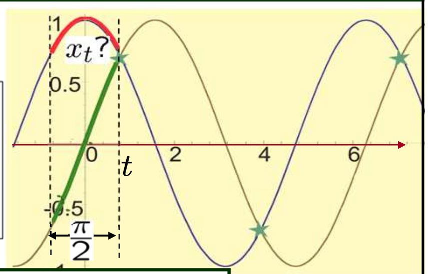


Simple example (cont'd)

$\dot{x}(t) + x(t-h) = 0$ $\quad h = \frac{\pi}{2}$

(Shimanov's notation , 1960)

$$\begin{aligned} \dot{x}(t) &= f\,(x_t, t, u_t)\,, & t &\geq t_0, \\ x_t(\theta) &= x(t+\theta), & -h &\leq \theta \leq 0, \\ u_t(\theta) &= u(t+\theta), & -h &\leq \theta \leq 0, \\ x(\theta) &= \varphi(\theta), & t_0 - h &\leq \theta \leq t_0, \end{aligned}$$



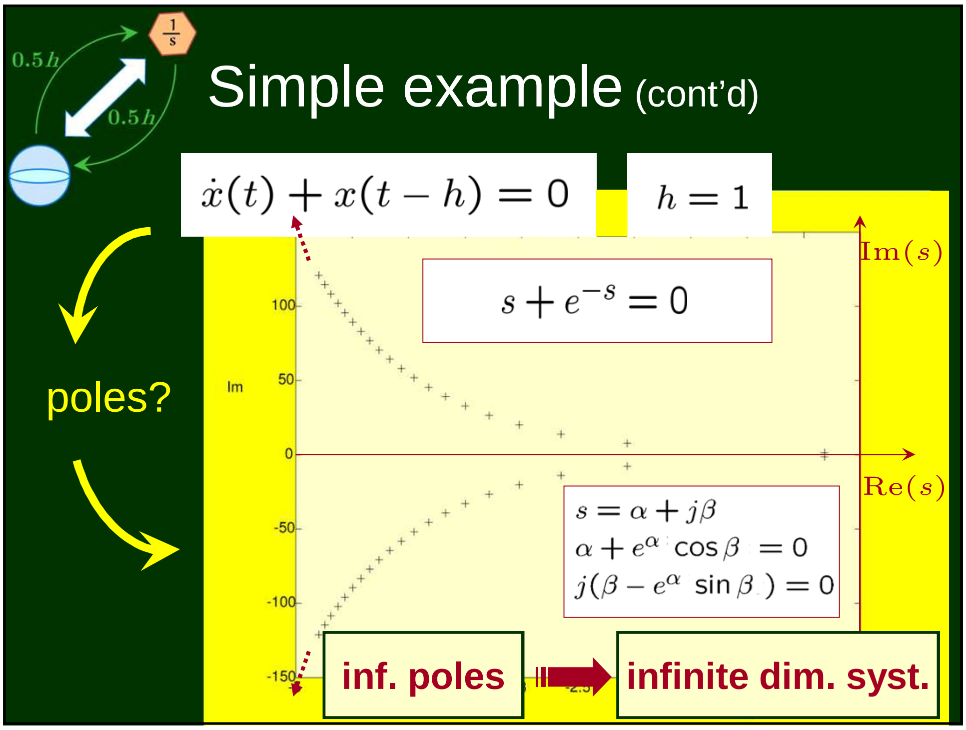
→ « state » notion ?

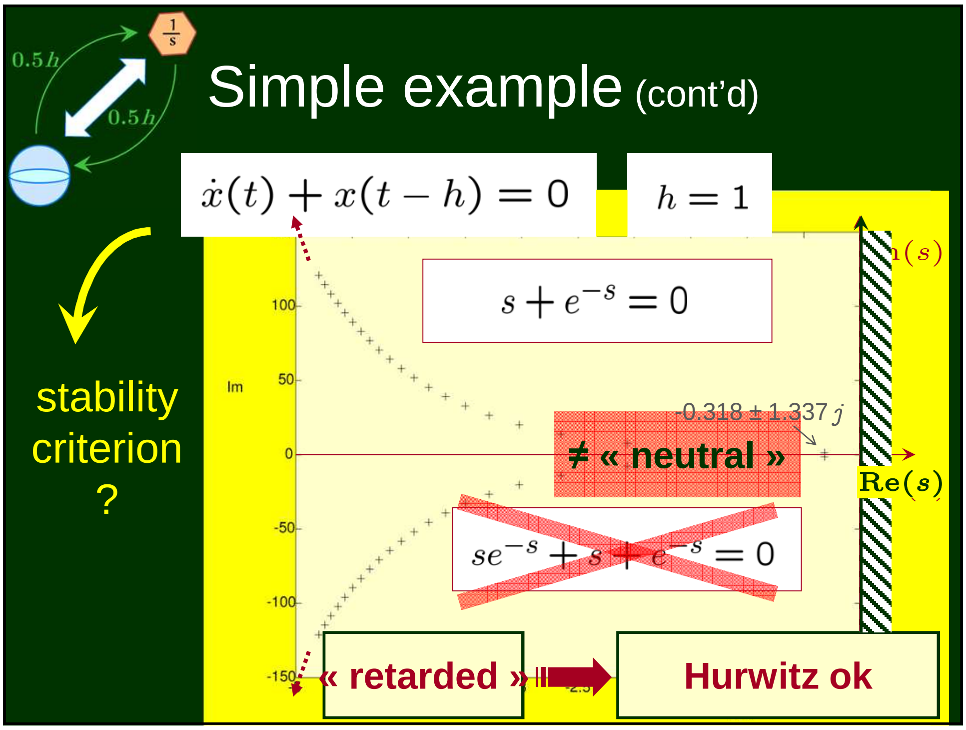
a variable $\phi(t)$ generating a unique solution starting at instant t

function x_t = state at time t

vector $x(t) = x_t(0)$ solution at t

⇒ function x_t FDE ⇒ **infinite dim. syst.**

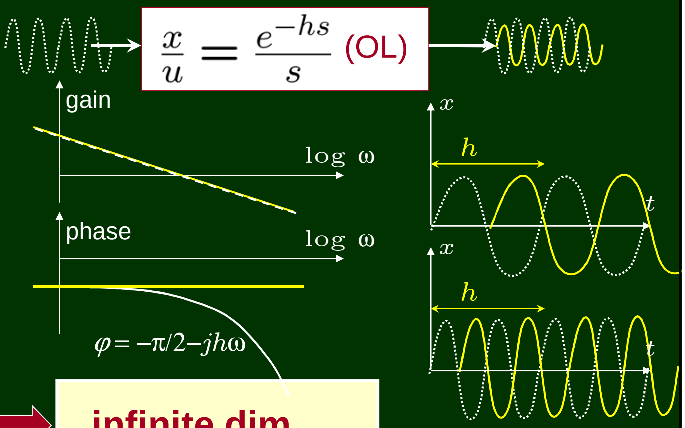

Simple example (cont'd)
$\dot{x}(t) + x(t-h) = 0$
$h = 1$
$s + e^{-s} = 0$
poles?
$\mathrm{Im}(s)$
$\mathrm{Re}(s)$
$s = \alpha + j\beta$
$\alpha + e^{\alpha}\cos\beta = 0$
$j(\beta - e^{\alpha}\sin\beta) = 0$
inf. poles
infinite dim. syst.
Im
100
50
0
-50
-100
-150


Simple example (cont'd)
$\dot{x}(t) + x(t-h) = 0$
$h = 1$
$s + e^{-s} = 0$
stability criterion ?
-0.318 ± 1.337 j
≠ « neutral »
$se^{-s} + s + e^{-s} = 0$
$\mathrm{Re}(s)$
« retarded »
Hurwitz ok
Im
100
50
0
-50
-100
-150

Simple example (cont'd)

$$\dot{x}(t) + x(t-h) = 0$$

frequency behavior ?
(Bode, open loop)



phase → ∞

phase -∞ ⇒ **infinite dim. ...**

Simple example (the end!)

$$\dot{x}(t) + x(t-h) = 0$$

to sum up...

delay ⇒ strong influence on stability
fonctionnal state
infinite nb of poles (Hurwitz OK, Routh no)
strong phase displacement (→ –∞)

and, up to now, it was not that complicated

constant delay
linear, scalar system « 1^{rst} order »

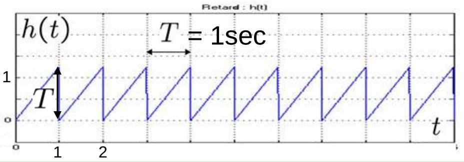
is it the same for variable delays $h(t)$?

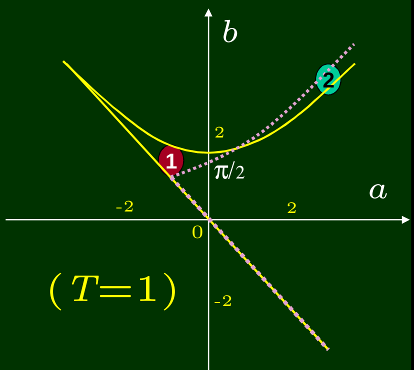
a counter-example...

(counter-)example with variable delay

$$\dot{x}(t) = -ax(t) - bx(t-h(t)) \qquad (1)$$

$$h(t) = t - kT \quad \text{for} \quad kT < t \leq (k+1)T$$





is asymptotically stable iff (yellow zone) :

$$\left|(1+\tfrac{b}{a})e^{-aT} - \tfrac{b}{a})\right| < 1 \quad \text{si } a \neq 0$$

$$|1 - bT| < 1 \quad \text{si } a = 0$$

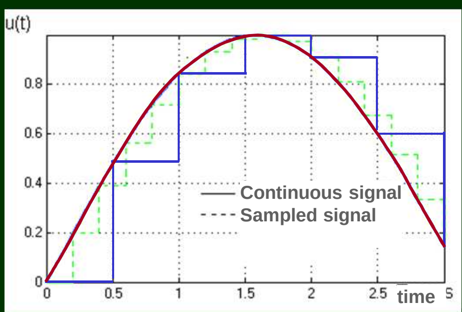
and, for $h =$ cst ∈ [0,1] iff (pink zone)

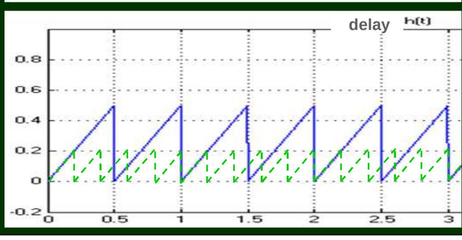
1. stable $h(t)$<1 - unstable h=cst<1
2. unstable $h(t)$<1 - stable h=cst<1

OK, but does such a delay $h(t)$ happen? another example...

Sampled systems: an interesting idea…

Mikheev et al. 88, Sobolev et al. 89. Aström et al. 92





Fridman, Seuret, Richard - *Automatica* 2004

Sampled and hold signal

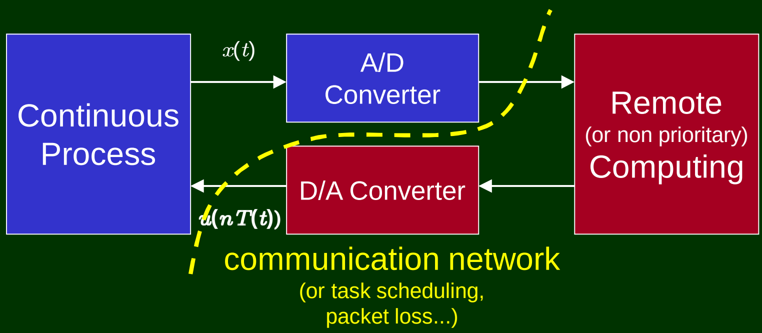
(depicted for a constant period)

⇕

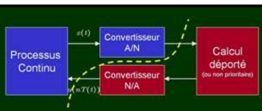
Delayed signal with variable $h(t)$

$$u(t) = u_d(t_k) = u_d(t - [t - t_k]) = u(t - h(t))$$

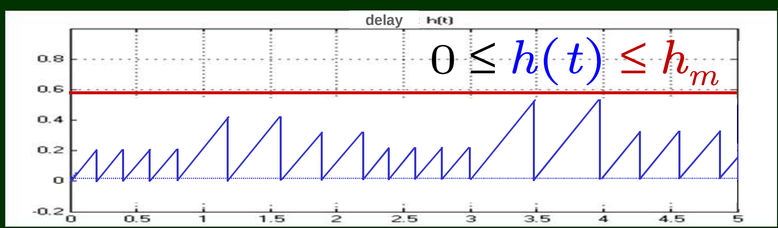
…with application to aperiodic sampling



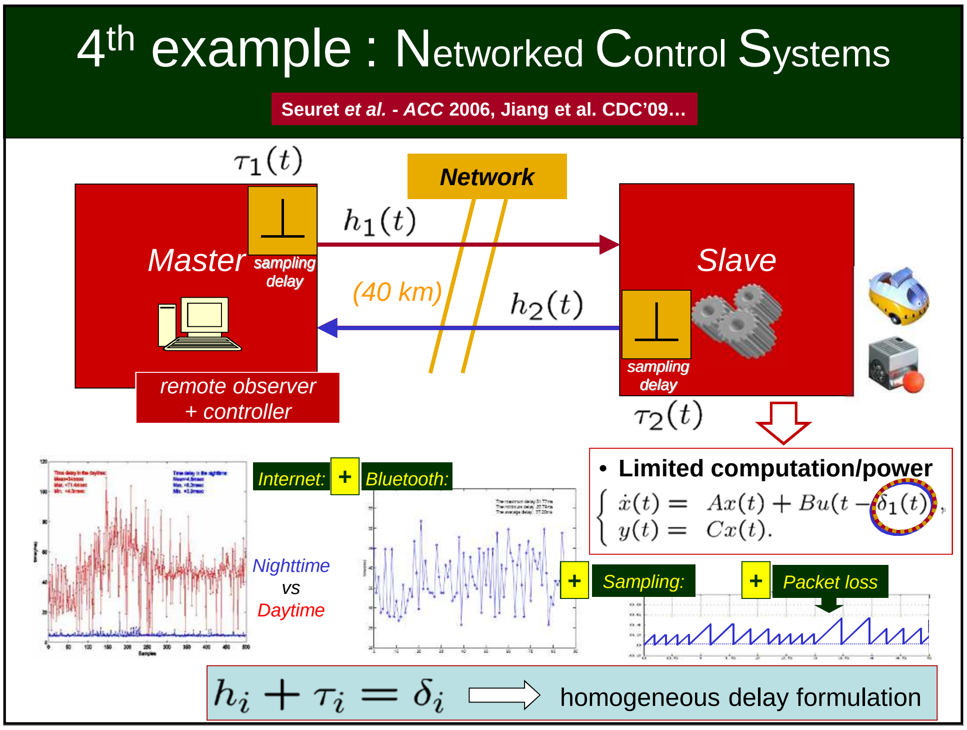
$$u(t) = g(x(nT(t)))$$

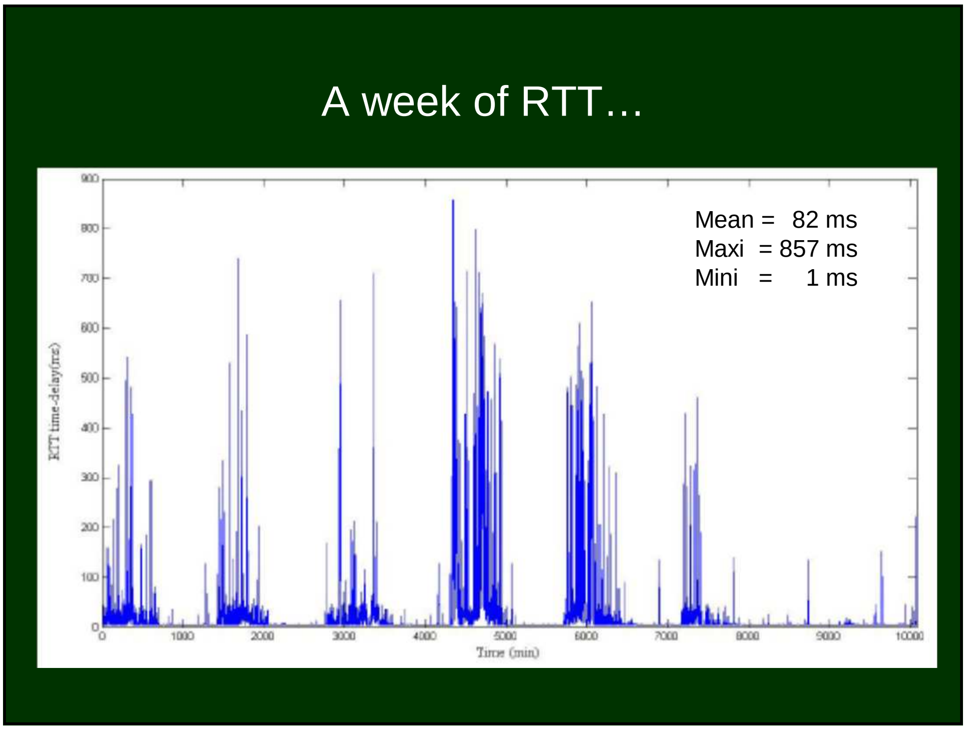


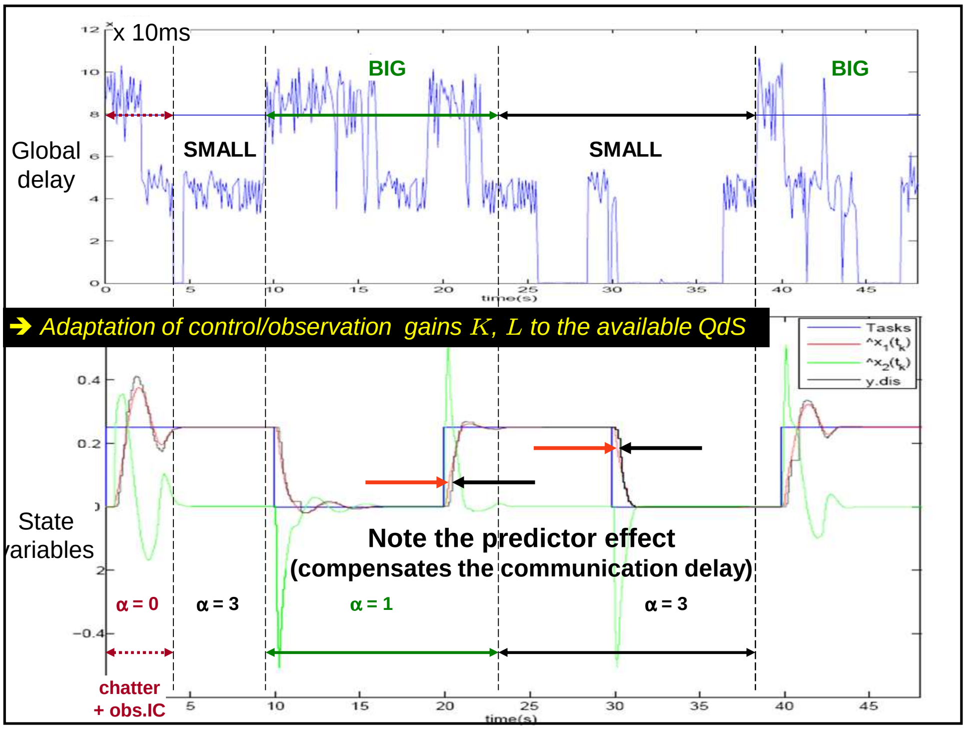
Another statement of the same problem...



- Influence of the *maximum* sampling period h_m
- Application of Fridman's criterion $\mathrm{d}h/\mathrm{d}t \le 1$


4th example : Networked Control Systems
Seuret et al. - ACC 2006, Jiang et al. CDC'09...
$\tau_1(t)$
Network
$h_1(t)$
Master
sampling delay
(40 km)
$h_2(t)$
Slave
sampling delay
remote observer + controller
$\tau_2(t)$
Limited computation/power
$\dot{x}(t) = Ax(t) + Bu(t - \delta_1(t)),$
$y(t) = Cx(t).$
Internet:
+
Bluetooth:
Nighttime
vs
Daytime
+
Sampling:
+
Packet loss
$h_i + \tau_i = \delta_i$
homogeneous delay formulation


A week of RTT…
Mean = 82 ms
Maxi = 857 ms
Mini = 1 ms
RTT time-delay(ms)
Time (min)
0
100
200
300
400
500
600
700
800
900
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000

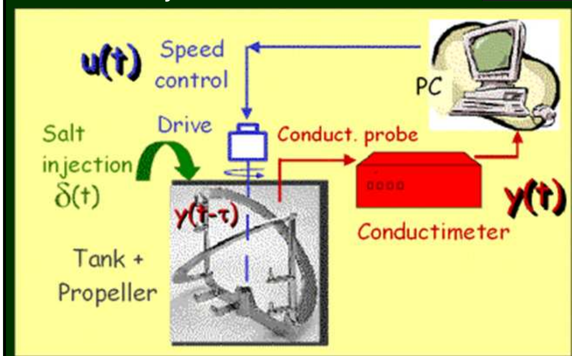


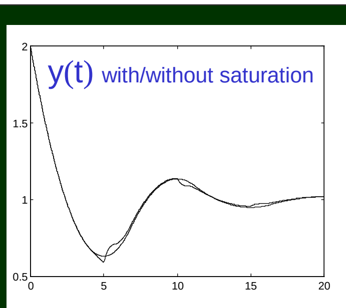
Goodies (dedicated to Thomas Erneux)

Mixing tank system (with total recycle)

- Control = delay = k(motor speed)$^{-1}$
- Non flat system

$$T(u(t))\dot{y}(t) = y(t - h(u(t)) - y(t),$$





An arrangement of ideal zones with shifting boundaries as a way to model mixing processes in unsteady stirring conditions in agitated vessels

J.-Y. Dieulot[a,*], N. Petit[b], P. Rouchon[b], G. Delaplace[c]
Chemical Engineering Science 60 (2005) 5544–5554

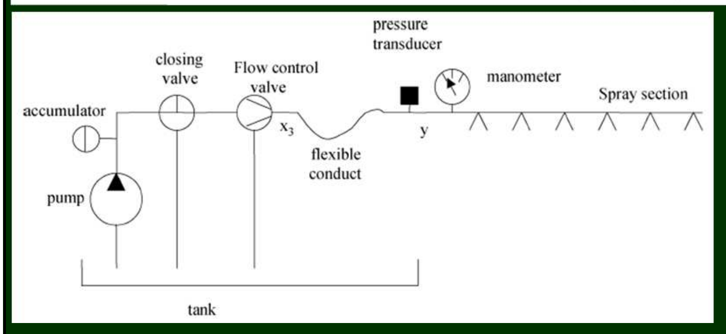
Dieulot, J. Y., & Richard, J. P. (2001). Tracking control of a nonlinear system with input-dependent delay. In *40th IEEE CDC'01* (*Conference on decision and control*), Orlando, FL, December 2001.

Goodies (2) (still dedicated to Thomas Erneux)

Design of a Pressure Control System With Dead Band and Time Delay

Jan Anthonis, Alexandre Seuret, Jean-Pierre Richard, and Herman Ramon

IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, VOL. 15, NO. 6, NOVEMBER 2007



→ deadzone (dry friction on angle x_1)
\+ delay (measured pressure y)
\+ sign functions (control)

$$\sqrt{y} = \frac{\alpha x_1(t-h)+\beta}{x_1(t-h)+\gamma}, \quad 0 \le h_{\min} \le h \le h_{\max}.$$

OK, that's enough examples,

let's go back to general questions

Contents

Distinctive features of TDS?

Illustrative examples

- 1^{rst} example (remote crtl.) → basic notions (stability, state, inf. dim.)
- 2^{nd} example : variable delay → counter-example
- 3^{rd} example : sampling → delay for modelling
- 4^{th} example : Networked Control System (master-slave)

Cauchy's problem for TDS

- Notion of solution
- Lipschitz-type condition
- And if the delay can vanish…

Stability and Lyapunov

- the LTI case
- 1^{rst} Lyapunov (small states)
- small delays
- 2^{nd} Lyapunov

Some words about identification

~~Mathematics~~ ;)
(just a little bit)

Cauchy's problem

(existence and unicity of solution for a TDS)

- Notion of solution
- Lipschitz-type condition
- And if the delay can vanish…

Cauchy pb. (1/4)

2.3 Notion of solution

System (S) : $\dot{x}(t) = f(t, x(t), x(t - \tau(t)))$

with $x(t) \in \mathbb{R}^n$, and $0 \leq \tau(t) \leq \tau$

Let $\varphi : [-\tau, 0] \longrightarrow \mathbb{R}^n$ be an arbitrary map.

Definition: *A map $x(t) : [t_0 - \tau, t_0 + b) \longrightarrow \mathbb{R}^n$ s.t.*
1) $x(t_0 + s) = \varphi(s)$, for all s in $[-\tau, 0]$;
2) x is continuous over $[t_0, t_0 + b)$;
3) x satisfies (S) over $[t_0, t_0 + b)$ ($\dot{x}$ right-hand, Dini)
is called a solution of (S) with initial value φ at t_0.
If only one map satisfies these 3 points, then the solution is unique.

Remark: There is a weaker notion of solution, where
2)$\longrightarrow$ x *absolutely continuous* function over $[t_0, t_0 + b)$
3)$\longrightarrow$ x satisfies (S) *almost everywhere* on $[t_0, t_0 + b)$

Cauchy pb. 2/4

2.4 Existence and uniqueness of solutions

For system (S) with $0 < \delta \leq \tau(t) \leq \tau_m$:

$$\dot{x}(t) = f(t, x(t), x(t - \tau(t))).$$

Consequence of the step method:
Given a continuous map $\varphi \in \mathcal{C}$, if the ODE

$$\dot{x}(t) = f_\varphi(t, x(t)) \equiv f(t, x(t), \varphi(t - \tau(t)))$$

has a (unique) solution, then there exists a (unique) solution of (S) with initial condition φ

From there, using classical Cauchy-Lipschitz cond$^{\text{tions}}$:
$\longrightarrow$Conditions of existence and uniqueness (I):
If f is a continuous map and satisfies a local Lipschitz condition in x,

$$\|f(t, x_2, y) - f(t, x_1, y)\| \leq K \|x_2 - x_1\|,$$

then for any initial condition $\varphi \in \mathcal{C}$, (S) has a unique solution, depending continuously on f and φ.

Cauchy pb. 3/4

If the delay can become zero, $0 \le \tau(t) \le \tau_m$ the step method does not apply anymore

$\Rightarrow$ need of a general framework: FDEs [Myshkis 49]

$$(RFDE): \quad \dot{x}(t) = F_R(t, x_t) \qquad \text{(retarded type)}$$

Conditions of existence and uniqueness (II):

If F_R is a continuous map with local-Lipschitz cond. in its second (functional) argument, i.e.

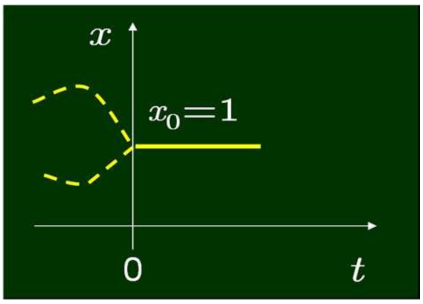
$$\|F_R(t, \varphi_2) - F_R(t, \varphi_1)\| \le K \|\varphi_2 - \varphi_1\|_{\mathcal{C}}, \dots$$

then for any initial condition $\varphi \in \mathcal{C}([-\tau, 0], \mathbb{R}^n)$, (RFDE) has a unique solution, depending continuously on F_R and φ.

This condition is also necessary for previous system (S) (discrete, bounded, nonzero delay)

Cauchy pb. 4/4

Remark



Even if unicity holds, different solutions may coincide after a finite time. For instance:

$$\dot{x}(t) = -x(t-\tau)[1 - x(t)],$$

$$x(t, \varphi) = 1 \quad (\forall t \ge 0)$$

for any $\varphi \in \mathcal{C}([-\tau, 0], \mathbb{R})$ such that $\varphi(0) = 1$.

(non-unicity of the trajectory reversion)

Contents

Distinctive features of TDS?

Stability

- the LTI case
- 1rst Lyapunov (small states)
- small delays
- 2nd Lyapunov

Stability : the LTI case

Theorem:

A linear time-invariant system (thus,with a constant delay):

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-h),$$

is globally asymptotically stable iff all its characteristic roots:

$$\det(sI - A_0 - e^{-hs} A_1) = \ldots 0$$

are in the strict left half plane .

Exemple 1. *Considérons l'équation* $\dot{x}(t) = -x(t-1)$. *Son équation caractéristique est* $s + e^{-s} = 0$, *dont les solutions* $s = \alpha \pm j\beta$ *sont en nombre infini. Le système n'est donc pas dégénéré. Ici,* $s = -0.318 \pm 1.337j$ *est une estimation de la paire de racines de plus grande partie réelle : il y a donc stabilité asymptotique*[2]. *Par contre, le cas suivant est dégénéré et instable :*

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \\ 0 & -\frac{1}{2} & 0 \end{pmatrix} x(t) + \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} x(t-h),$$

$$\det(sI - A_0 - e^{-hs} A_1) = \ldots \; s(s^2 - 1).$$

Stability : 1^{rst} method of Lyapunov

« approximation of the small deviations »

$$\dot{x}(t) = \sum_{i=0}^{k} A_i x(t-h_i) + q(t, x_t) \qquad (5.8)$$

$$q(t, x_t) = q(t, x(t), x(t-\tau_1(t)), \ldots, x(t-\tau_k(t)),$$

$$h_0 = 0, \; h_i = \text{constantes}, \; \tau_j(t) \in [0, \tau_i] \text{ continues},$$

$$\|u_i\| \leq \varepsilon \Rightarrow \|q(t, u_0, \ldots, u_k)\| \leq \beta_\varepsilon(\|u_0\| + \ldots + \|u_k\|),$$

avec β_ε = constante pour ε donné, β_ε uniformément décroissante vers 0 quand $\varepsilon \to 0$. L'approximation au premier ordre est définie par :

$$\dot{z}(t) = \sum_{i=0}^{k} A_i z(t-h_i). \qquad (5.9)$$

Théorème 4. [26] *Si le système linéarisé (5.9) est asymptotiquement stable, alors* $z = 0$ *l'est aussi pour (5.8). Si (5.9) a au moins une racine caractéristique à partie réelle positive, alors* $z = 0$ *est instable pour (5.8).*

Stability: LTI with small delays

« approximation of small delays »

$$\dot{z}(t) = \sum_{i=0}^{k} A_i z(t - h_i). \quad (5.9)$$

$$\dot{x}(t) = \sum_{i=0}^{k} A_i x(t - h_i) + q(t, x_t) \quad (5.8)$$

Le résultat précédent peut être utilement complété par une *approximation des petits retards*, résultat de nature qualitative obtenu par continuité des racines caractéristiques de (5.9) vis-à-vis des retards h_i.

Théorème 5. [26] *Si $A = \sum_{i=0}^{k} A_i$ est de Hurwitz (respectivement, instable), alors pour des valeurs suffisamment faibles des retards h_i, la solution nulle $z = 0$ est asymptotiquement stable (respectivement, instable) pour (5.9) et donc (5.8). Si, sur les n valeurs propres de A, $n-1$ ont des parties réelles strictement négatives et la $n-$ième est nulle, alors, pour des valeurs suffisamment faibles des h_i, $z = 0$ est stable pour (5.9) et donc pour (5.8).*

Stability: LTI with small, single delay

quantification of a « small » admissible delay

$$\frac{dz(t)}{dt} = A_0 z(t) + A_1 z(t - h), \quad (5.10)$$

qui, pour un retard nul, devient :

Sufficient condition

$$\frac{dz(t)}{dt} = (A_0 + A_1)\, z(t). \quad (5.11)$$

Théorème 6. [40] *Si le système à retard nul (5.11) est asymptotiquement stable et si P est la matrice solution de l'équation de Liapounov (où Q est une matrice réelle définie positive [117]) :*

$$(A_0 + A_1)^T P + P (A_0 + A_1) = -Q^T Q, \quad (5.12)$$

alors (5.10) est asymptotiquement stable pour tout retard $h \in [0, h_{\max}]$:

$$h_{\max} = \frac{1}{2} \left[\lambda_{\max}(B^T B)\right]^{-\frac{1}{2}}, \quad \text{avec } B = Q^{-T} A_1^T P (A_0 + A_1) Q^{-1}. \quad (5.13)$$

Lyapunov for TDS

Another result by Kolmanovskii & Myshkis for LTI TDS (1999)

$$\dot{x}(t) = Ax(t-h)$$

(also in nonlinear $\dot{x}(t) = f(t, x(t-h))$)

« dissipative systems »

$V(x_t) = ||x(t)||$ (some norm)

$||A||$ = associated matrix norm,

$\gamma(A)$ = logarithmic norm ("measure").

$$\gamma(A) < -h||A||^2 \implies \text{expon. stable, } e^{-\omega t}$$

ω : solution of $\omega = -\gamma(A) - h||A||^2 e^{2\omega h}$

Lyapunov's direct method for TDS

ODE :

$\dot{x}(t) = -ax(t)$ ➡ $V(x(t)) = x^2(t) > 0$

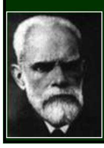
$\dot{V}(x(t)) = -2\ ax^2(t) < 0\ldots$ etc.

FDE :

$\dot{x}(t) = -ax(t) - bx(t-h)$

$V(x(t)) = x^2(t)$ (« usual » quadratic)

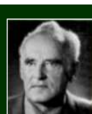
$\dot{V}(x(t)) = -2\ [ax^2(t) + bx(t)x(t-h)] \leq \ldots$? (cross terms: $bx(t)x(t-h)$)



➔ need of delay-dedicated methods :

1) Lyapunov-Razumikhin functions (not here)

2) Lyapunov-Krasovskii functionals



Lyapunov for TDS

an illustration of the Lyapunov-Krasovskii approach

$$\dot{x}(t) = -ax(t) - bx(t-h)$$

$$V(x_t) = x^2(t) + |b| \boxed{\int_{-h}^{0} x^2(t+s)ds}$$ (quad + integral)

$$\dot{V}(x_t) = -2x(t)[ax(t) + bx(t-h)] + |b|\underline{[x^2(t) - x^2(t-h)]}$$

$$\leq -2(a - |b|)x^2(t) \quad \ldots \; \dot{V}(x_t) < 0 \text{ if } |b| < a$$

NB: LK-functionals were used in the above NCS/sampled data proofs
(under a much more general form)

A bit more general LKF…

$$(S) \quad \dot{x}(t) = Ax(t) + Bx(t-\tau), \text{ avec } x(t) \in \mathbb{R}^n.$$

Fonctionnelle : $\mathcal{V}(\varphi) = \varphi^T(0)P\varphi(0) + \int_{-\tau}^{0} \varphi^T(s)S\varphi(s)\,\mathrm{d}s$
avec $P,\ S \succ 0$.

$$\Rightarrow \boxed{\dot{\mathcal{V}}(x_t) = y^T(t)Qy(t)}$$

avec $Q = \begin{bmatrix} A^TP + PA + S & PB \\ B^TP & -S \end{bmatrix}$ et $y(t) = \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}$

$\Rightarrow$ Stabilité asymptotique i.d.r. si $Q \prec 0$ (LMI)

and a way to delay-dependent stabilty…

Hyp. : (S) asymp. stable pour $\tau = 0 \Rightarrow \mathcal{A} = A + B$ Hurwitz

Problème

Chercher une borne τ^* t.q. stab. asympt. $\forall \tau \leq \tau^*$.

Idée

Transformation du modèle à l'aide de la formule de Leibniz-Newton :

$$x(t) - x(t-\tau) = \int_{-\tau}^{0} \dot{x}(t+s)\,\mathrm{d}s$$

(S)$\Rightarrow$

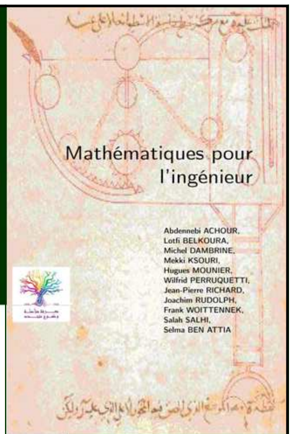
$$\dot{x}(t) = (A+B)x(t) - B\int_{t-\tau}^{t} (Ax(s) + Bx(s-\tau))\,\mathrm{d}s$$

+ many more general LKFs

see the textbook *Mathématiques pour l'Ingénieur*

ISBN : 978-9973-0-0852-7 (Tunisie) *385 pages*, 2009

pdf available on request…



$$\dot{x}(t) = \sum_{i=1}^{m} A_i x(t-h_i). \tag{6.47}$$

$$A = \sum_{i=1}^{m} A_i, \quad A_{ij} = A_i A_j, \quad h_{ij} = h_i + h_j, \quad h = \sum_{i=1}^{m} h_i. \tag{6.48}$$

Théorème 6.5.8. *Le système (6.47) est asymptotiquement stable si, pour deux matrices symétriques et définies positives R, Q, il existe une matrice définie positive P solution de l'équation de Riccati :*

$$A^T P + PA + mRh + P\sum_{i,j=1}^{m} h_i A_{ij} R^{-1} A_{ij}^T P = -Q. \tag{6.49}$$

Démonstration : on choisit la fonctionnelle $V = V_1 + V_2$, $V_1 = x^T(t)Px(t)$, $V_2 = \sum_{i,j=1}^{m} \int_{h_j}^{h_{ij}} ds \int_{t-s}^{t} x^T(\tau)Rx(\tau)d\tau$, conduisant à $\dot{V} = -x^T(t)\,Qx(t) - \sum_{i,j=1}^{m} \int_{t-h_j}^{t-h_{ij}} [Rx(\theta) + A_{ij}^T Px(t)]R^{-1}[Rx(\theta) + A_{ij}^T Px(t)]^T d\theta$. ■

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- Lipschitz-type condition
- And if the delay can vanish…

Stability and Lyapunov

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… Few last words about identification

Some words on our current research in identification/estimation

ALIEN : Algebra for Identification & Estimation

http://www.inria.fr/lille/ & http://hal.inria.fr/lab/ALIEN/

INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE
INRIA
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a foreword on identifiability

(dedicated to Gilles Millerioux…)

INTERNATIONAL JOURNAL OF ROBUST AND NONLINEAR CONTROL
Int. J. Robust Nonlinear Control 2003; **13**:857–872 (DOI: 10.1002/rnc.850)

Adaptive identification of linear time-delay systems

Y. Orlov[1,*,†], L. Belkoura[2,‡], J.P. Richard[3,§] and M. Dambrine[3,¶]

$$\dot{x}(t) = \Sigma_{i=0}^{r}[A_i x(t-\tau_i) + B_i u(t-\tau_i)], \tag{1}$$

$$\dot{x}(t) = A(\lambda)x(t) + B(\lambda)u(t), \tag{6}$$

$$y(t) = C(\lambda)x(t) \tag{7}$$

over the ring $\mathbf{R}[\lambda]$ of polynomials in a vector variable $\lambda = (\lambda_1, ..., \lambda_k)^T$

In matrix terms system (1) is weakly controllable iff for some $z \in \mathbf{C}^k$.

$$rank\ \left[B(z) \mid A(z)B(z) \mid \ldots \mid A^{n-1}(z)B(z)\right] = n \tag{9}$$

Definition 1 *System (1) is said to be identifiable if there exists a control input $u(t)$ such that the identity $x(t) \equiv \hat{x}(t)$ results in*

$$r = \hat{r},\ \tau_i = \hat{\tau}_i,\ A_i = \hat{A}_i,\ B_i = \hat{B}_i\ for\ i = 0, \ldots, r,$$

regardless of a choice of the initial functions $\varphi(\theta), \hat{\varphi}(\theta)$. In that case the identifiability is said to be enforced by the control input $u(t)$.

Theorem 1 *The time-delay system (1) is identifiable if and only if it is weakly controllable. Moreover, if (1) is weakly controllable then the identifiability can be enforced by any sufficiently nonsmooth control input $u(t)$.*

see also
by the same authors:

IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. 47, NO. 8, AUGUST 2002

On Identifiability of Linear Time-Delay Systems

… and, more general (convolutions):

Available online at www.sciencedirect.com
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Automatica 41 (2005) 505–512
automatica
www.elsevier.com/locate/automatica

Identifiabilty of systems described by convolution equations☆

Lotfi Belkoura*

Throughout this paper, we assume stable linear and time invariant systems with input $u(.) \in \mathbb{R}^q$ and output $y(.) \in \mathbb{R}^p$. The models considered are those that can be described by one the following convolution equations:

$$P * y = Q * u, \qquad (1)$$

$$P * z = u, \quad y = Q * z. \qquad (2)$$

Identification results: up to now, linear systems

→ Adaptive (continuous) techniques

- Diop, Kolmanovskii, Moraal, vanNieuwstadt – Control Eng.Pract. 9, 2001
  "Preserving stability/performance when facing an unknown time delay."
- Orlov, Dambrine, Belkoura, Richard - IJNRC 13, 2003
  (see above)
- Gomez, Orlov, Kolmanovskii – Automatica 43(12) 2007
  "On-line identification of SISO linear time-invariant delay systems from output measurements"

→ Nonsmooth techniques (VSS)

- Drakunov, Perruquetti, Richard, Belkoura, - Ann. Reviews in Control, 30(2) 2006
  "Delay identification in time-delay systems using variable structure observers"

→ Algebraic techniques (distributions)

- Belkoura, Richard, Fliess - Automatica 45(5) 2009
  "Parameters estimation of systems with delayed and structured entries"

The idea of ALIEN

(Fliess, Sira-Ramirez ESAIM COCV 2003)

Basic example, no-delay case

$$\dot{y}(t) = ay(t) + u(t) + \gamma$$

a : unknown parameter;
γ : constant perturbation

$$s\hat{y}(s) = a\hat{y}(s) + \hat{u}(s) + y_0 + \frac{\gamma}{s}$$ (y_0 : initial condition)

- elimination of γ:

$$\frac{d}{ds}\left[s\left\{s\hat{y}(s) = a\hat{y}(s) + \hat{u}(s) + y_0 + \frac{\gamma}{s}\right\}\right]$$

$$\Rightarrow 2s\hat{y}(s) + s^2\hat{y}'(s) = a\left(s\hat{y}'(s) + \hat{y}(s)\right) + s\hat{u}'(s) + \hat{u}(s) + y_0$$

ALIEN's idea, Ctn'd

- estimation of a: $(y_0 = 0)$

$$s^{-\nu}[2s\hat{y}(s) + s^2\hat{y}'(s) = a(s\hat{y}'(s) + \hat{y}(s)) + s\hat{u}'(s) + \hat{u}(s)]; \quad \nu > 0$$

$$y'(s) = \frac{dy(s)}{ds} = \mathcal{L}(-ty(t))$$

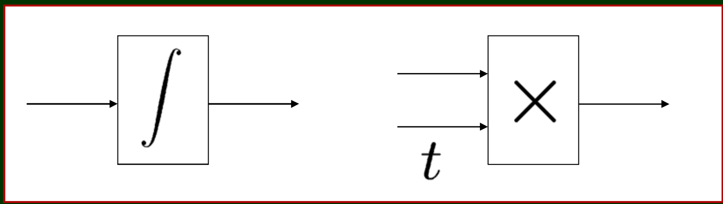
$$a = \frac{2\int_0^t d\lambda \int_0^\lambda y(\tau)d\tau - \int_0^t \tau y(\tau)d\tau + \int_0^t d\lambda \int_0^\lambda \tau u(\tau)d\tau - \int_0^t d\lambda \int_0^\lambda d\sigma \int_0^\sigma u(\tau)d\tau}{\int_0^t d\lambda \int_0^\lambda d\sigma \int_0^\sigma y(\tau)d\tau - \int_0^t d\lambda \int_0^\lambda \tau y(\tau)d\tau}$$

$\nu = 3$

- ☐ t may be very small → fast estimation
- number ν of integrations → averaging role

ALIEN's idea, Ctn'd

$$a = \frac{2\int_0^t d\lambda\int_0^\lambda y(\tau)d\tau - \int_0^t \tau y(\tau)d\tau + \int_0^t d\lambda\int_0^\lambda \tau u(\tau)d\tau - \int_0^t d\lambda\int_0^\lambda d\sigma\int_0^\sigma u(\tau)d\tau}{\int_0^t d\lambda\int_0^\lambda d\sigma\int_0^\sigma y(\tau)d\tau - \int_0^t d\lambda\int_0^\lambda \tau y(\tau)d\tau}$$

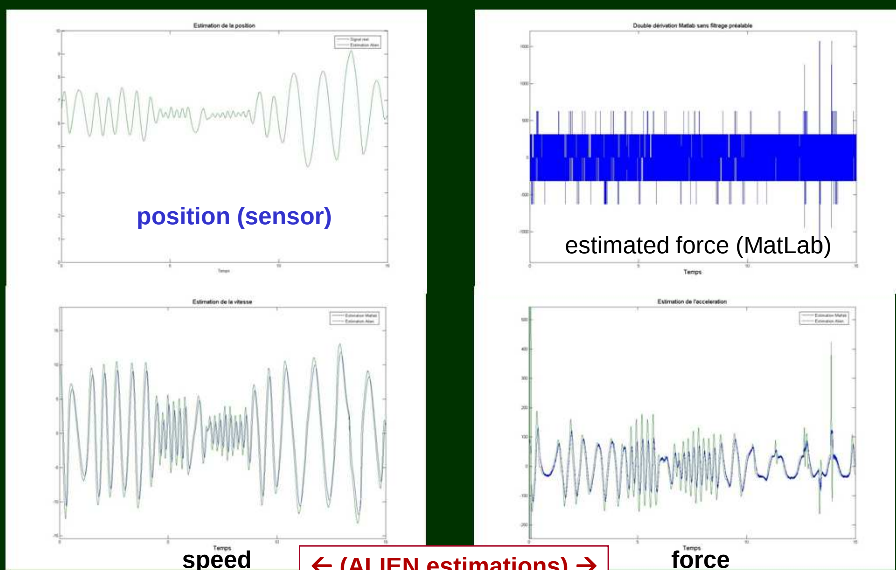


- parameters or states obtained via iterative integrations (or, more generally, low-pass filters)
- noise = fast fluctuations

ALIEN's idea, Ctn'd

Same approach works for derivative estimation

[Mboup, Join, Fliess, Numer. Algor. 2009]



ALIEN's idea, Ctn'd

... and it works for delay estimation

[Belkoura, Richard, Fliess, Automatica 2009]

$$\dot{y}(t) + ay(t) = y(0)\delta + \gamma_0 H + bu(t-\tau)$$ (basic example)

1rst results: simulation

$y(0) = 0.3, a = 2, \tau = 0.6,$
$\gamma_0 = 2, b = 1, u_0 = 1.$

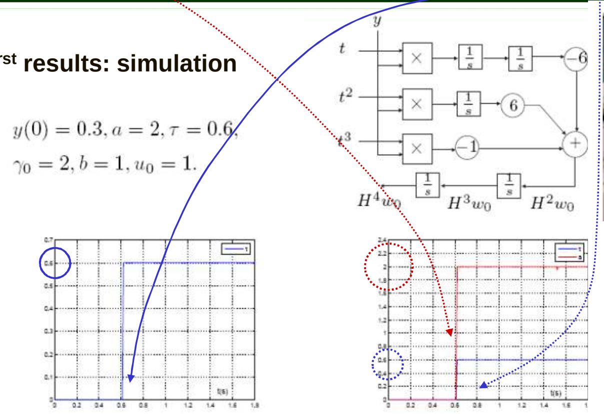
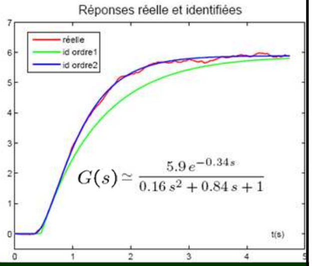


Figure 1: Identification du retard τ et identification simultanée de a et τ

$$\tau = \frac{H^k\left(t^3y^{(2)} + at^3y^{(1)}\right)}{H^k\left(t^2y^{(2)} + at^2y^{(1)}\right)}, \quad t > \tau.$$

2) real process (simple)



ALIEN's idea, Ctn'd

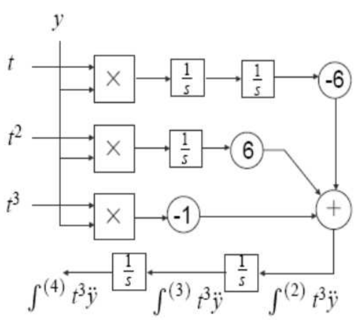
The integrations are performed using the integration by part formula, avoiding any derivative in the algorithm.

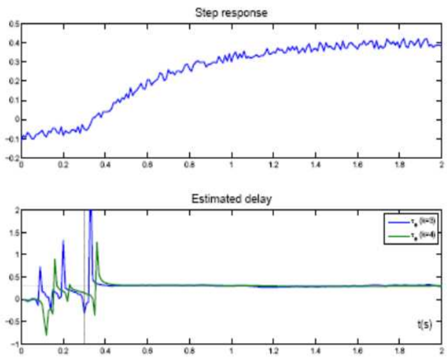
$$\text{Ex:} \quad \int^{(k)} t^3\,\ddot{y} = -6\int^{(k)} ty + 6\int^{(k-1)} t^2y - \int^{(k-2)} t^3y. \qquad (6)$$

Remark: Filters may be used instead of integrals.

$$\tau = \frac{H^k\left(t^3y^{(2)} + at^3y^{(1)}\right)}{H^k\left(t^2y^{(2)} + at^2y^{(1)}\right)}, \quad t > \tau.$$

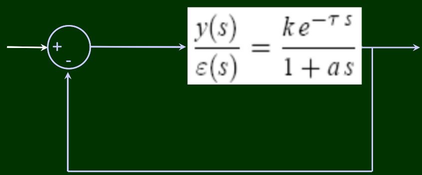
Partial realization scheme and simulation results.



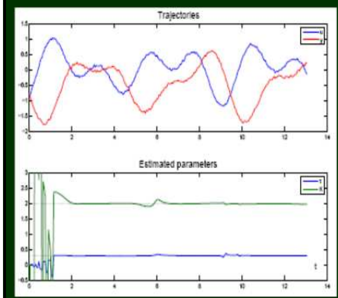


ALIEN's idea, Ctn'd

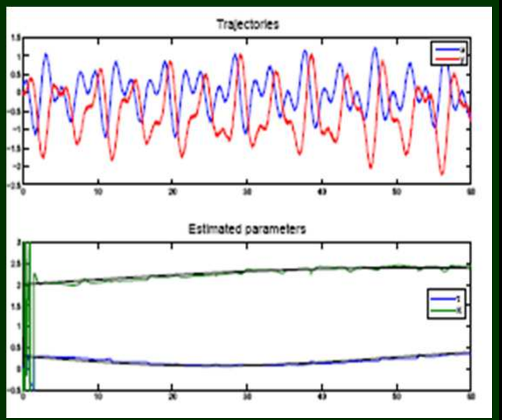
... and may work for closed loop, variable delay estimation



$$\frac{y(s)}{\varepsilon(s)} = \frac{k\,e^{-\tau s}}{1 + a\,s}$$



constant parameters



slowly varying parameters

Some general references

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