A polytopic approach for state-dependent sampling

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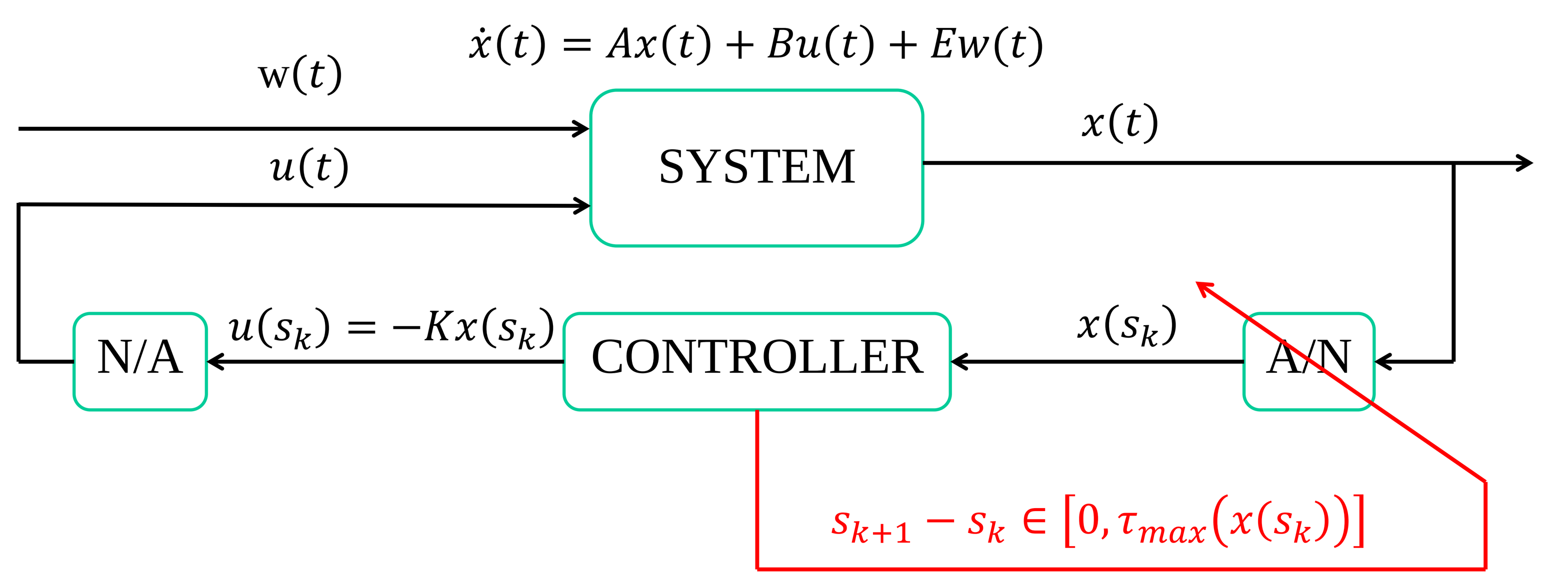
Abstract

This work aims at decreasing the number of sampling instants in state feedback control for perturbed linear time invariant systems. The approach is based on linear matrix inequalities obtained thanks to Lyapunov-Razumikhin stability conditions and convexification arguments that guarantee the exponential stability for a chosen decay-rate. First, the method enables to perform a robust stability analysis regarding time-varying sampling and to maximize a lower-bound estimate of the maximal allowable sampling interval, by computing the adequate Lyapunov-Razumikhin function. Then, it makes it possible to design a state-dependent sampling control scheme that enlarges even further the maximal allowable sampling intervals.

System description & Goals

Dynamic control of the sampling instants for sampled-data systems



Sampling control loop

State dependent sampling map $\tau_{max}: \mathbb{R}^n \longrightarrow \mathbb{R}_+$

Continuous LTI sampled-data system

$$\dot{x}(t) = Ax(t) + Bu(t) + Ew(t), \forall t \in \mathbb{R}_+$$
$$u(t) = -Kx(s_k), \forall t \in [s_k, s_{k+1})$$

Assumption on the perturbation

$$\|w(t)\|_2 \leq \sqrt{W}\|x(s_k)\|_2$$

Sampling law

$$s_{k+1} - s_k = \tau(s_k, x(s_k)) \equiv \tau_k \in [0, \tau_{max}(x(s_k))], \forall k \in \mathbb{N}$$

Goals :

- Sample as few times as possible
- Design offline the sampling map τ_{max} allowing maximal sampling intervals
- Maximize the lower bound of the sampling map
- Ensure exponential stability for a given decay rate β

Method & Results

Consider scalars $\alpha > 1, \bar{\sigma} > 0, W \geq 0, 0 < \beta \leq \frac{\ln(\alpha)}{2\bar{\sigma}}$, such that $\|\tau_{max}\|_{\infty} \leq \bar{\sigma}$.
Denote $x(t) \equiv \varphi_{\tau_{max}}(\sigma, x)$, $\quad x \equiv x(s_k)$, $\quad \sigma \equiv t - s_k$.

Lyapunov-Razumikhin β-stability condition:

If there exists a candidate Lyapunov function $V(x) = x^T Px$ such that
$$\forall x \in \mathbb{R}^n, \forall \sigma \in [0, \tau_{max}(x)],$$
$$\dot{V}(\varphi_{\tau_{max}}(\sigma, x)) + 2\beta V(\varphi_{\tau_{max}}(\sigma, x)) \leq 0$$
whenever $\alpha V(\varphi_{\tau_{max}}(\sigma, x)) \geq V(x)$, then the system is β-stable.

Lemma (matrix version of LR β-stability condition):

If there exist $P \succ 0$ and $\varepsilon \geq 0$ such that $\forall x \in \mathbb{R}^n, \sigma \in [0, \tau_{max}(x)]$,
$$\begin{bmatrix} \Lambda(\sigma)x + J_w(\sigma) \\ x \\ w(\sigma) \end{bmatrix}^T \Omega \begin{bmatrix} \Lambda(\sigma)x + J_w(\sigma) \\ x \\ w(\sigma) \end{bmatrix} \leq 0,$$
with $\Omega = \begin{bmatrix} A^T P + PA + \varepsilon\alpha P + 2\beta P & -PBK & PE \\ * & -\varepsilon P & 0 \\ * & * & 0 \end{bmatrix}$,
$\Lambda(\sigma) = I + \int_0^{\sigma} e^{sA} ds\, (A - BK)$, *and* $J_w(\sigma) = \int_0^{\sigma} e^{(\sigma - s)A} Ew(s) ds$,
then the system is β-stable.

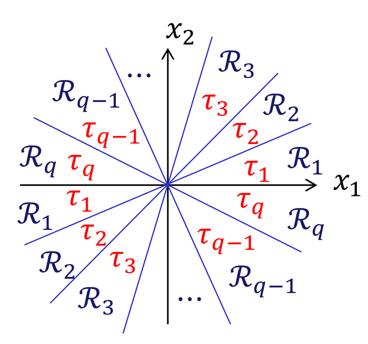
Theorem (condition depending only on sampled-state x and time σ):

If there exist $P \succ 0, \varepsilon \geq 0$, and (some additional parameters) such that some LMIs are satisfied and such that $\forall x \in \mathbb{R}^n, \sigma \in [0, \tau_{max}(x)]$,
$$x^T \phi(\sigma) x \leq 0,$$
(with some matrix function $\phi: [0, \tau_{max}(x)] \rightarrow \mathbb{R}^{n \times n}$), then the system is β-stable.

Reduction of the number of inequalities to verify by using :

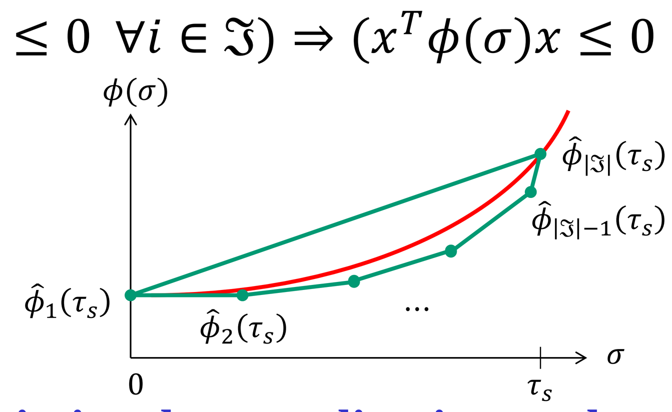
<u>A conic partition of the state space :</u>

The state is divided into a finite number of regions $\mathcal{R}_s$, $s \in \{1, \cdots, q\}$. Each region $\mathcal{R}_s$ is associated to one sampling interval τ_s.



<u>A convex embedding according to time:</u>

For each region $\mathcal{R}_s$, a convex polytope $Co\{\hat{\phi}_i(\tau_s), i \in \mathfrak{I} \text{ (finite)}\}$ is designed (using a Taylor Polynomial of ϕ) such that $\forall x \in \mathcal{R}_s$,
$$(x^T \hat{\phi}_i(\tau_s) x \leq 0 \;\; \forall i \in \mathfrak{I}) \Rightarrow (x^T \phi(\sigma) x \leq 0 \;\; \forall \sigma \in [0, \tau_s]).$$



Theorem (to maximize the sampling intervals τ_s for a given Lyapunov-Razumikhin function $V(x) = x^T Px$, $P \succ 0$) :

The system is β-stable if there exist scalars $\varepsilon_{i,s} \geq 0$ such that the LMIs
$$\hat{\phi}_i(\tau_s) + \varepsilon_{i,s} Q_s \preccurlyeq 0 \text{ are satisfied for all } i \in \mathfrak{I}, s \in \{1, \cdots, q\}.$$

Corollary (to compute the Lyapunov function V maximizing the lower bound $\tau^* = \inf_{x \in \mathbb{R}^n} \tau_{max}(x)$ of the sampling map τ_{max}) :

The system is β-stable for any time-varying sampling bounded by τ^* if there exists $P \succ 0, \varepsilon \geq 0$ (and additional parameters), such that the LMIs $\hat{\phi}_i(\tau^*) \preccurlyeq 0$ are satisfied for all $i \in \mathfrak{I}$.

Numerical example

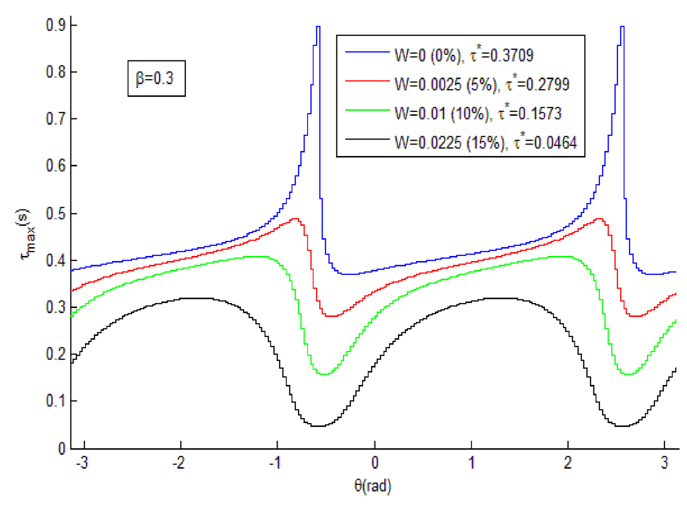
$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} x(t) - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & 4 \end{bmatrix} x(s_k) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} w(t)$$

	$\beta = 0$	$\beta = 0,1$	$\beta = 0,3$
$W = 0$ (0%)	0,5402s	0,4404s	0,3709s
$W = 0{,}0025$ (5%)	0,4975s	0,4092s	0,2799s
$W = 0{,}01$(10%)	0,4271s	0,3364s	0,1573s
$W = 0{,}04$ (20%)	0,2719s	0,1814s	-
$W = 0{,}09$ (30%)	0,1417s	0,0518s	-
$W = 0{,}16$ (40%)	0,0322s	-	-

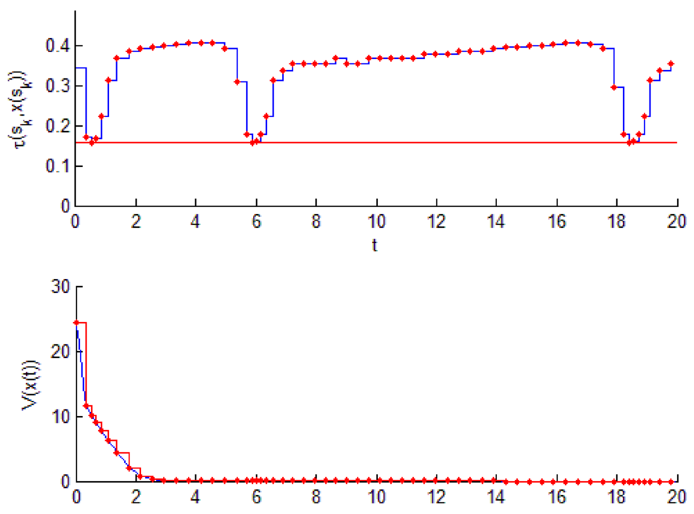
Maximum upper-bounds τ^* for time-varying sampling

	$\beta = 0$	$\beta = 0,1$	$\beta = 0,3$
Naghshtabrizi & al. [S&CL 2008]	0,2740s	-	-
Seuret [CDC 2009]	0,3122s	0,2795s	0,1778s
Fujioka [Automatica 2009]	0,3316s	-	-
Fridman [Automatica 2010]	0,4221s	0,3934s	0,3350s
Fiter & al.	0,5402s	0,4404s	0,3709s

Maximum upper-bounds τ^* for time-varying sampling, without perturbation



State dependent sampling functions for a decay-rate $\beta = 0{,}3$ and different W



Simulation results (sampling intervals and LRF) for $\beta = 0{,}3$ and $W = 0{,}01$ ($\|w(t)\|_2 \leq 10\%\|x(s_k)\|_2$)

Conclusion

- Design of a state dependent sampling function allowing to reduce the number of actuations for LTI systems.
- Offline computation of the Lyapunov-Razumikhin function V and the state dependent sampling map τ_{max} $\Rightarrow$ No additional online computation required!
- Maximization of the minimal sampling interval τ^* $\Longrightarrow$ Ensures large sampling intervals even in the worst case!

References

- <u>Robust stability with respect to sampling period variations</u>: Fridman [Automatica 2010], Seuret [CDC 2009], Fujioka [Automatica 2009]
- <u>Event-Triggered Control</u>: Tabuada [TAC 2007], Lunze & Lehman [Automatica 2010], Heemels & al. [IJC 2008]
- <u>Self-Triggered Control</u>: Mazo Jr. & al. [Automatica 2010], Anta & Tabuada [TAC 2010], Wang & Lemmon [TAC 2010], Velasco & al. [RTSS 2003]
- <u>State-Dependent Sampling</u>: Fiter & al. [Automatica 2012], Fiter & al. [CDC 2012]
- <u>Convex Embeddings</u>: Hetel & al. [ACC 2007], Heemels & al. [HSCC 2010]