Flow Control: TD #2

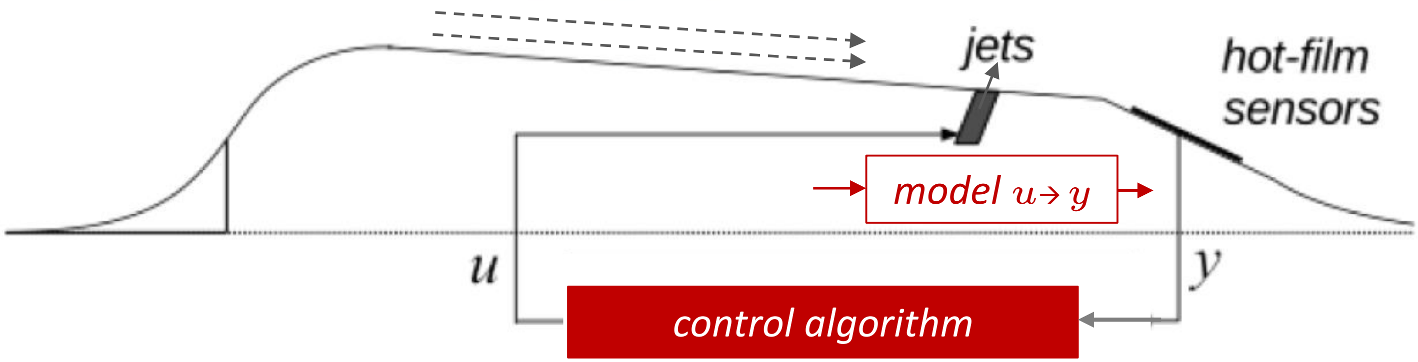
Overview

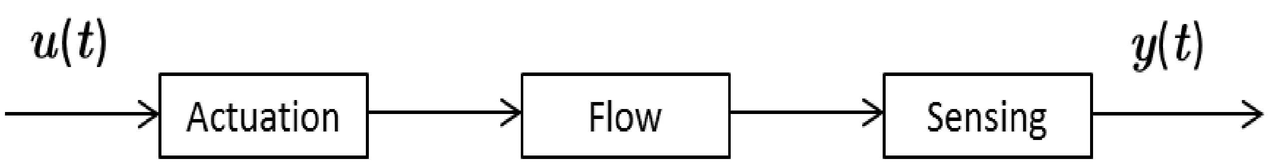
- General issues, passive *vs* active…
- Control issues: optimality and learning *vs* robustness and rough model
- Model-based control: linear model, nonlinear control
    - Linear model, identification
    - Sliding Mode Control
    - Delay effect
    - Time-delay systems
- Introduction to delay systems
    - Examples
    - Much a do about delay? Some special features + a bit of maths
    - Time-varying delay
- Model-based control: nonlinear model, nonlinear control
    - Overview of MF’s PhD: Sliding Mode Control
    - Application to the airfoil
    - Application to the Ahmed body (MF and CC’PhDs)

…

- Machine Learning and model-free control: + 4h with Thomas Gomez

Input – Output flow Control: linear case





… explain me how you're gonna do?

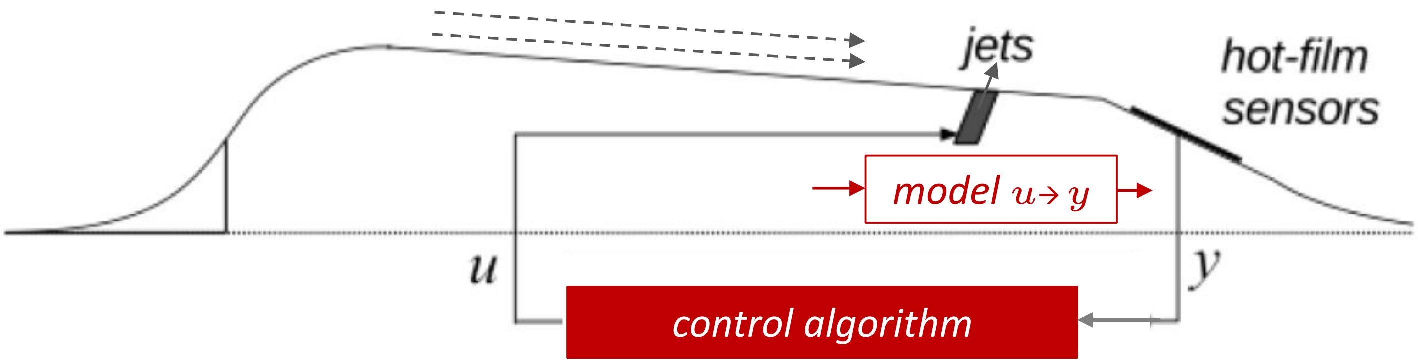
Flow Control

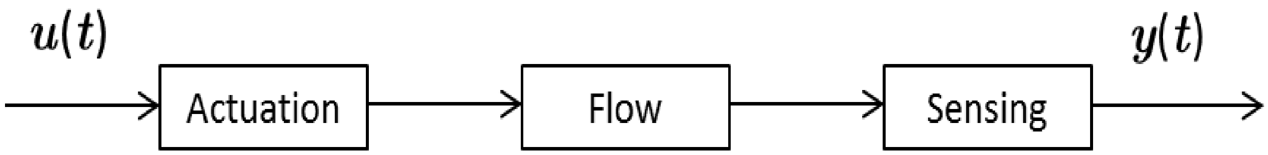
Control specification: a *memorandum*

- Tracking
  - structural properties: controllability, observability
  - set point tracking, trajectory tracking
- Stability
  - small initial perturbation => small perturbation for all
- Acuracy
  - small final gap (asymptotic property) ....
  - wrt some class of trajectories (t^n, $n = 1, 2, 3$...) ....
- Speed of convergence
  - asymptotic, exponential decay rate and time constant ..... ⇨ open-loop
  - finite-time, fixed-time .... ⇨ closed-loop
- Robustness
  - wrt external disturbances .... ⇨ closed-loop
  - wrt model error .... ⇨ closed-loop
- Low energetic cost
  - LQR optimal: minimize $\int_0^\infty (gap^2 + energy^2)dt$ .... ⇨ closed-loop
- Low price
  - technology (actuators, sensors, computing…)
  - design time

⇨ open-loop

Input – Output flow Control: linear case





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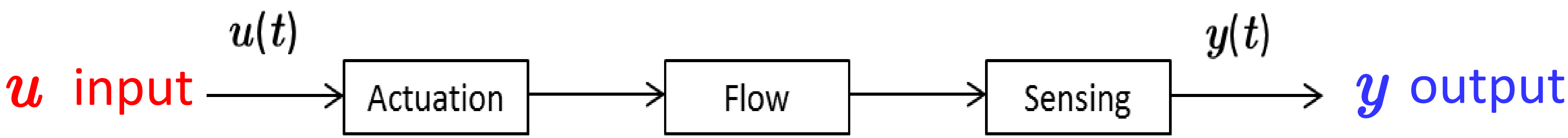
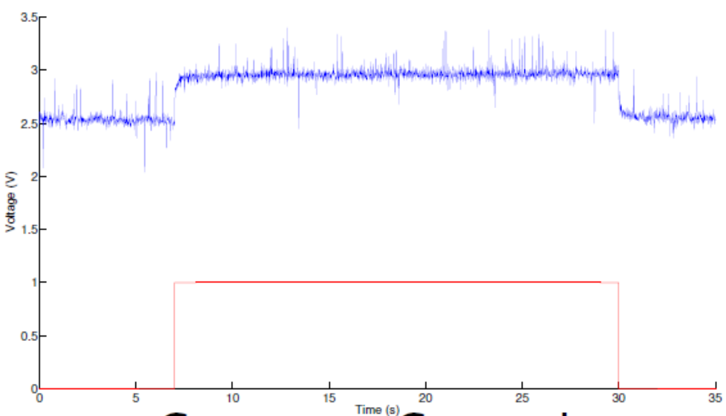
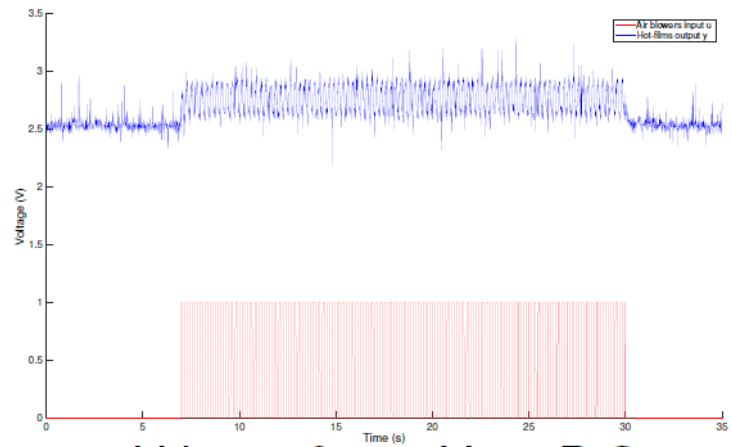
Input – Output flow Control: linear case

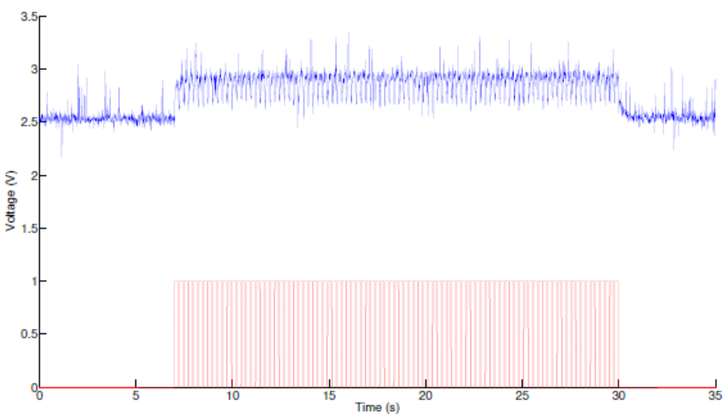




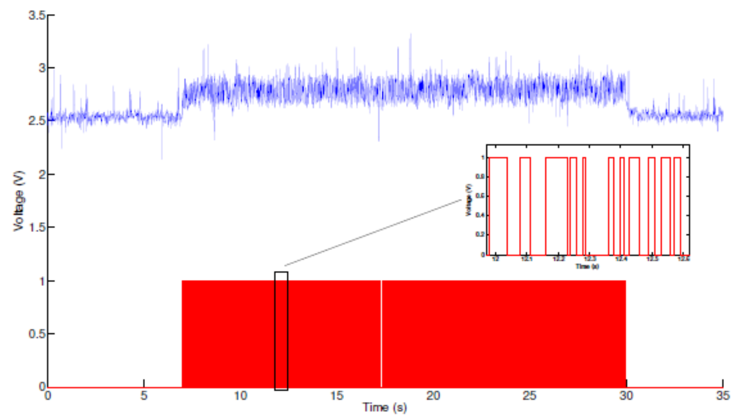
Constant Control



Square Wave, f = 4Hz , DC=50%

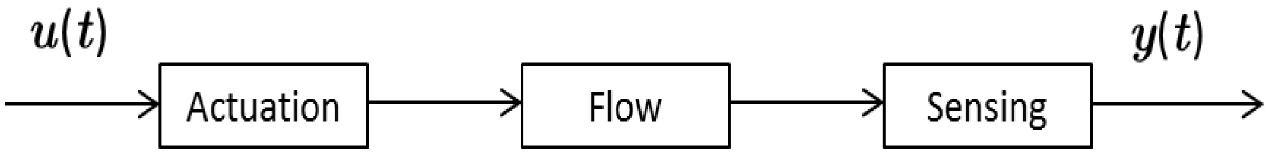


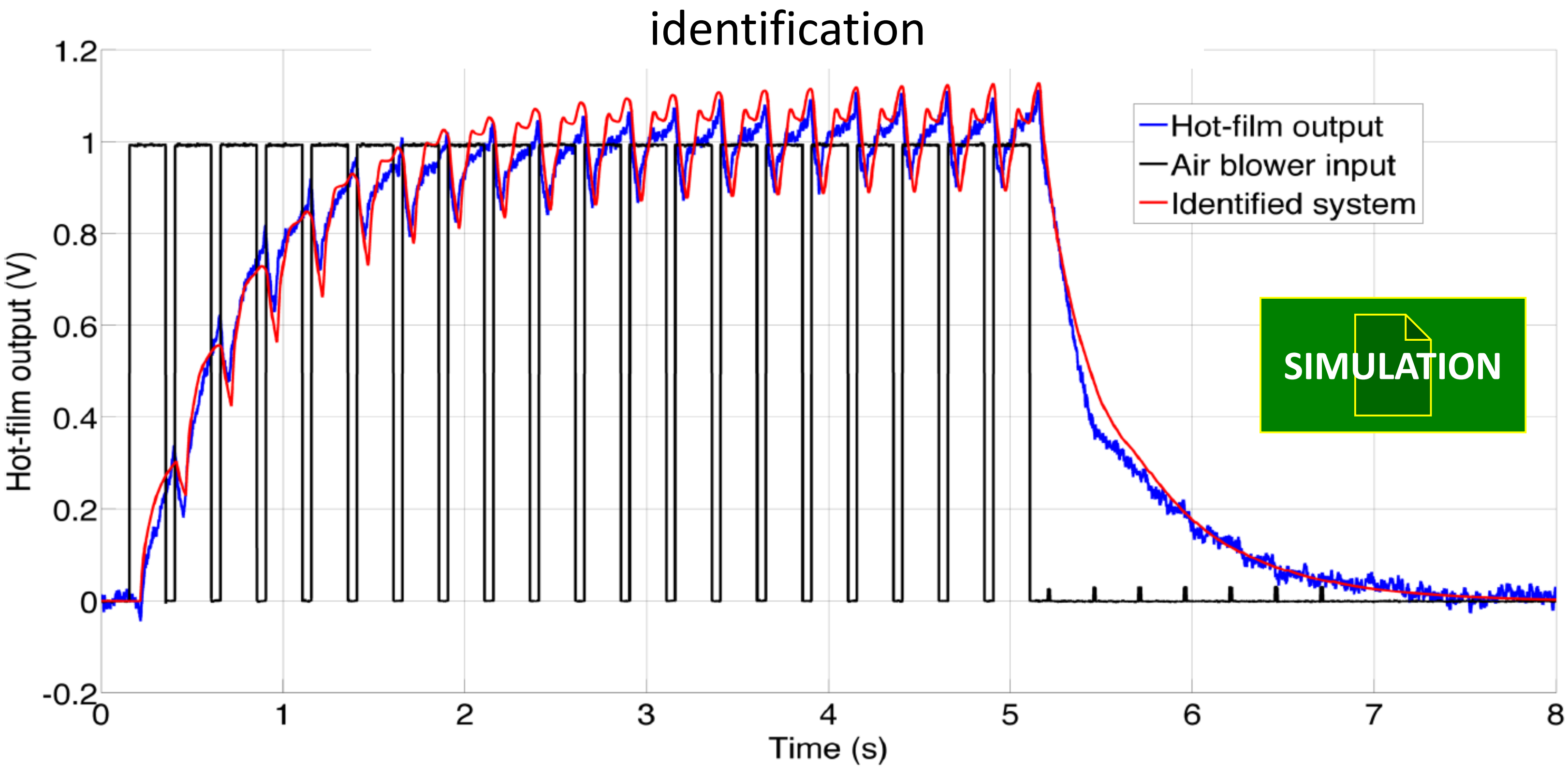
Square Wave, f = 4Hz , DC=80%



PRBS

Input – Output flow Control: linear case

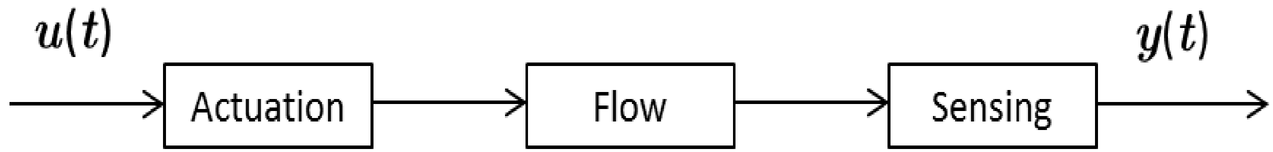




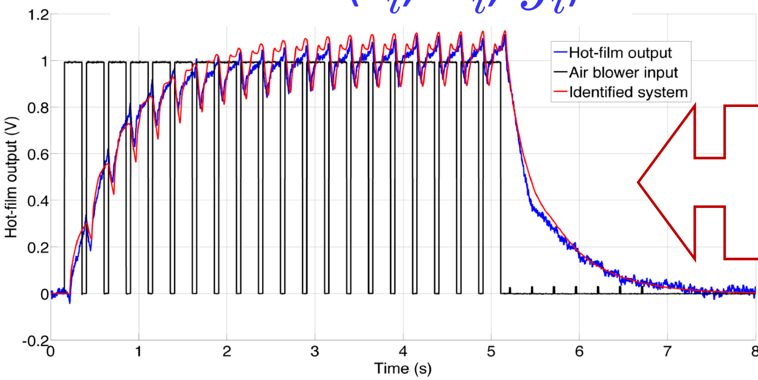
Propose some model.

$$\dot{y}(t) = -ay(t) + bu(t) \quad ?$$

Input – Output flow Control: linear case



data set (t_i, u_i, y_i)



How do you proceed with the identification ?

$$\dot{y}(t) = -ay(t) + bu(t)$$

Data (u_i, y_i) × Parameters (a, b) = computed $\dot{y}$

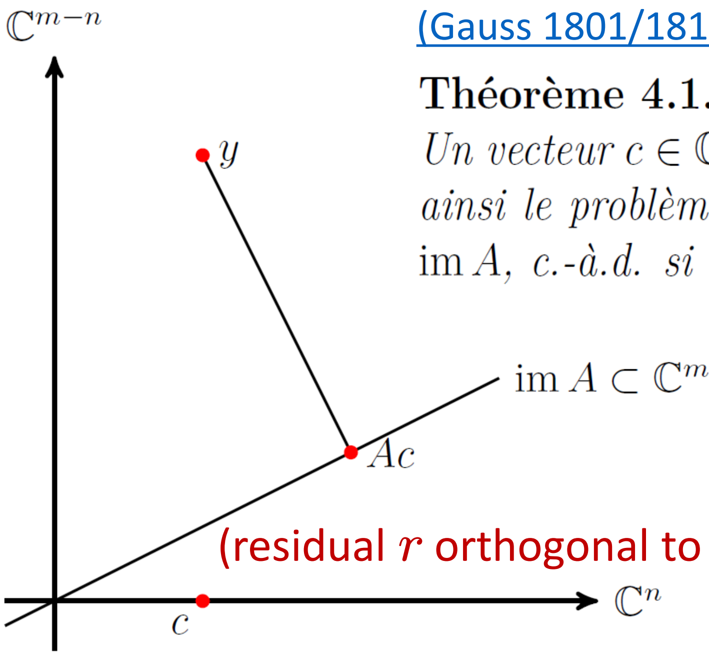
Data × Parameters = Data

$$\boxed{Ac = y}$$ → *find* c → *pseudoinverse* A^+ *of* A

$$A \in R^{m \times n}$$

$$m > n$$

Input – Output flow Control: linear case



(Gauss 1801/1810 – orbit of Céres $m=11$ $n=6$, Legendre 1805 – orbit of comets)

Théorème 4.1. *Soit une matrice $A \in \mathbb{C}^{m\times n}$, $m > n$, et une donnée $y \in \mathbb{C}^m$. Un vecteur $c \in \mathbb{C}^n$ minimise la norme des résidus $||r||_2 = ||y - Ac||_2$, résolvant ainsi le problèmes des moindres carrés, si et seulement si r est orthogonal à* $\operatorname{im} A$, *c.-à.d. si* $\boxed{A^* r = 0}$ *ou bien* $A^*(Ac) = A^* y$ *(équations normales)*

(residual r orthogonal to im A)

$$A^*(Ac) = A^* y$$

$A^* A \in \mathbb{C}^{n\times n}$ regular iff $\operatorname{rank} A = n$.

$$c = (A^*A)^{-1} A^* y$$

$$A^+ \triangleq (A^*A)^{-1} A^* \in \mathbb{C}^{n\times m}$$

pseudoinverse

Data x Parameters = Data

$$Ac = y$$

$A \in R^{m\times n}$

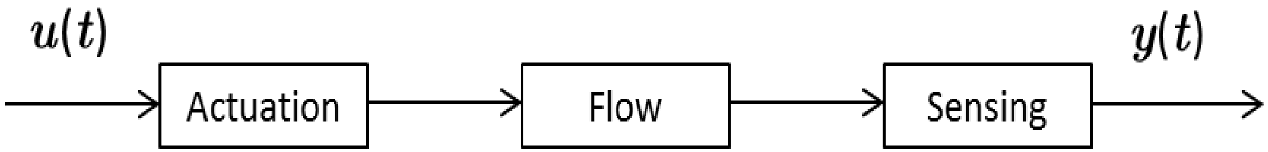
$m > n$

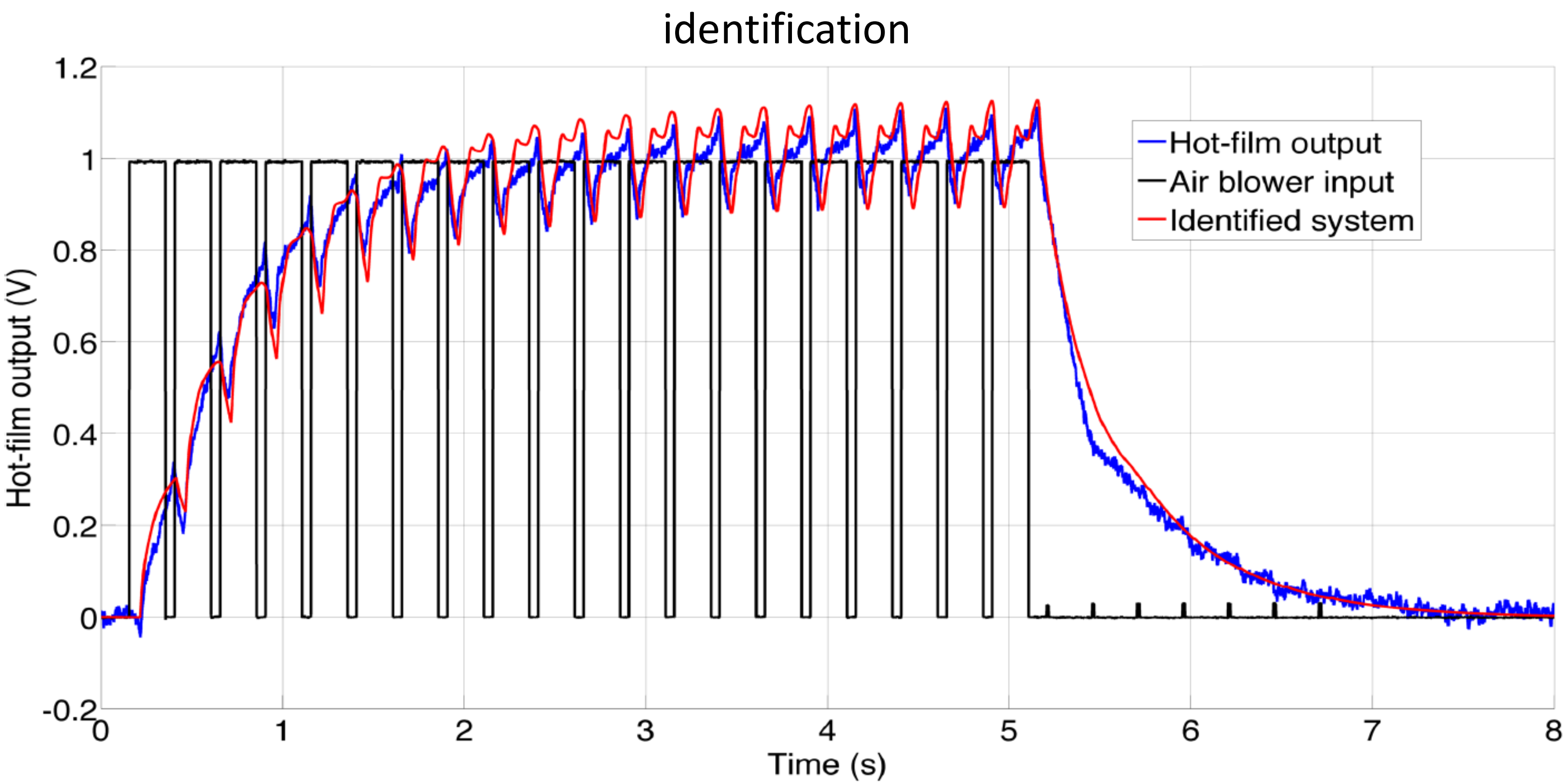
$$r \triangleq y - Ac$$

Residual to be minimized w.r.t. c (least squares)

$$||y - Ac||_2$$

Input – Output flow Control: linear case





$$\dot{y}(t) = -ay(t) + bu(t)$$

… with on/off control (actuation technology)

Input – Output flow Control: linear case

on/off control = *sliding mode control* (SMC)

Model

Consider the following model:

$$\dot{y}(t) = -ay(t) + bu(t) \quad (8)$$

with $a > 0$, $b > 0$, $u(t) \in \{0, 1\}$, $y(0) \in [0, y_{max}]$. The solution y is bounded for any input signal $u(t)$: $0 \leq y(t) \leq y_{max}$.
Control objective: track the setpoint y^*.

?

Input – Output flow Control: linear case

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Sliding surface

$$\Sigma = \{y : \sigma(y) = 0\}$$

with σ the sliding variable:

$$\sigma(y) = y - y^*$$

Input – Output flow Control: linear case

Control

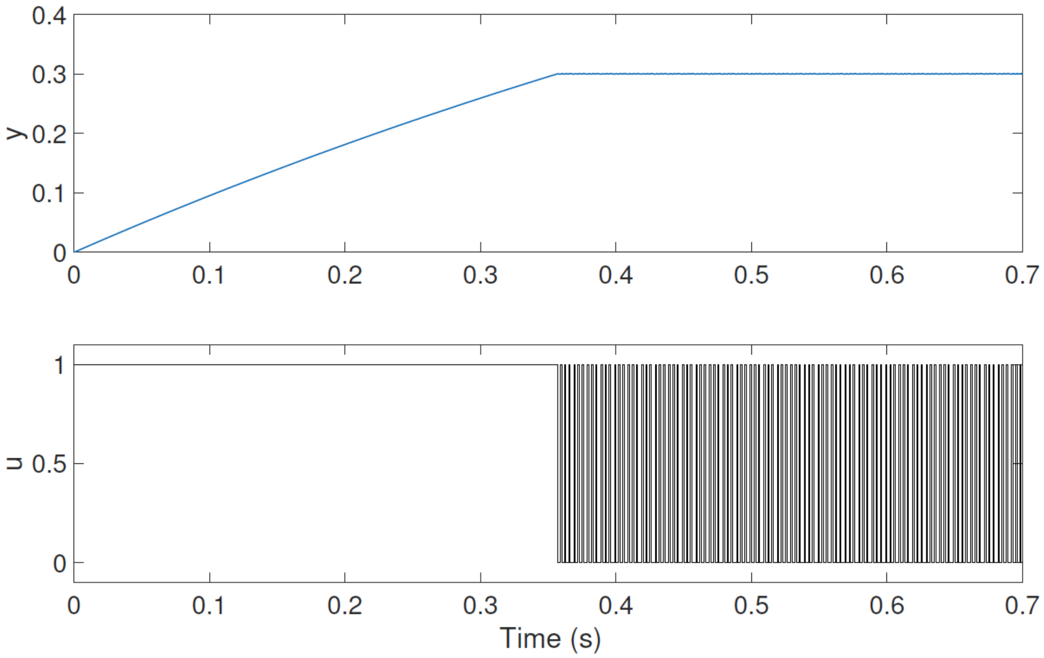
$$u(t) = \begin{cases} 1 & \text{if } \sigma(y) \leq 0 \\ 0 & \text{if } \sigma(y) > 0 \end{cases}$$

If $\dot{\sigma}(y)\sigma(y) < 0$ then the system reaches Σ in finite time (Utkin 1992) and stays on Σ: $\exists t^* > 0, \sigma(y) = 0$ for $t > t^*$

Reaching condition

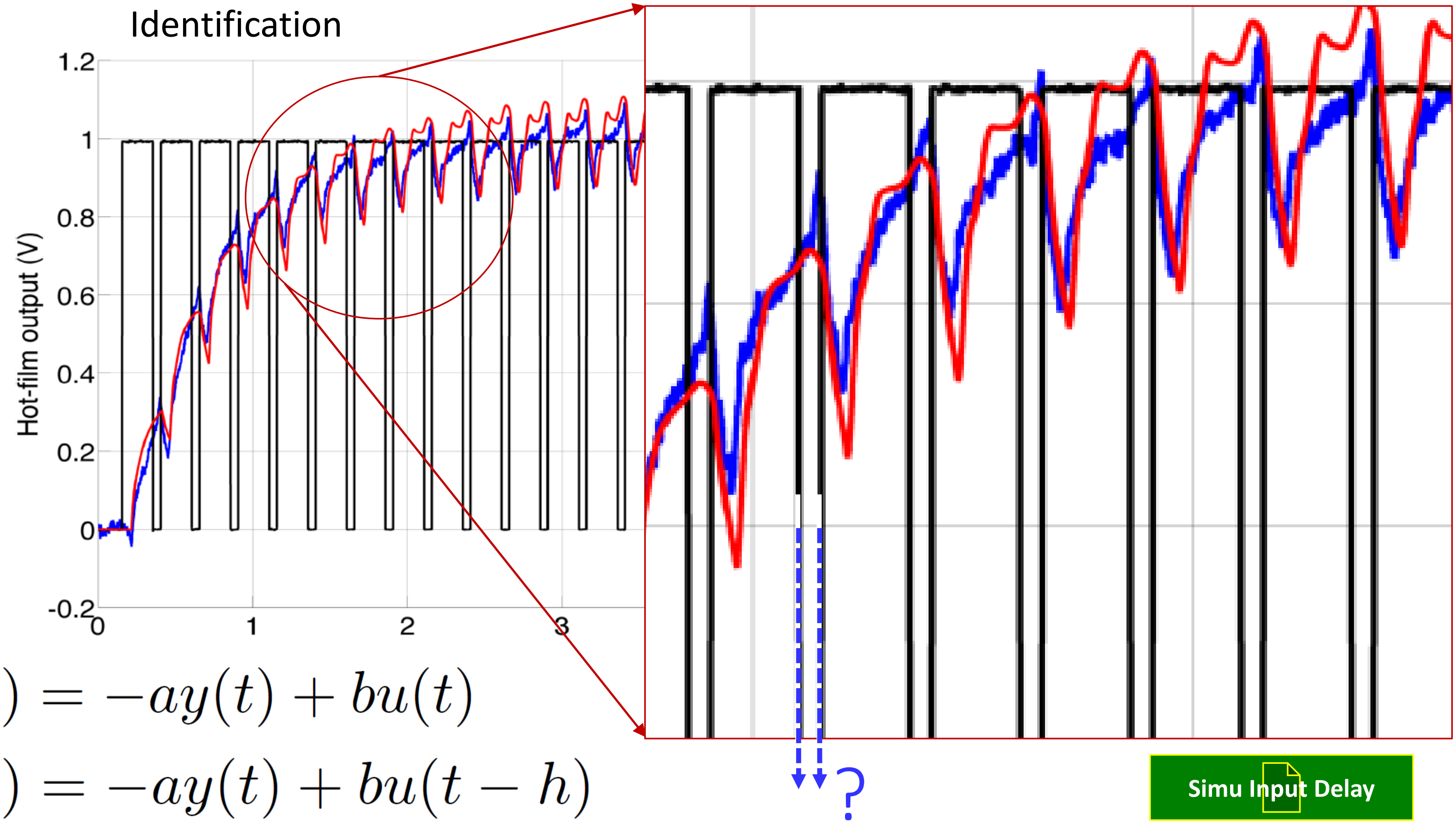
$$\begin{aligned} \dot{\sigma}(y)\sigma(y) &= \dot{y}(t)(y(t) - y^*) \\ &= (-ay(t) + bu(t))(y(t) - y^*) \\ &= \begin{cases} (-ay(t) + b)(y(t) - y^*) & \text{if } y(t) < y^* \\ -ay(t)(y(t) - y^*) & \text{if } y(t) > y^* \end{cases} \\ &< 0 \quad \text{as} \quad 0 \leq y(t) \leq y_{max} \end{aligned}$$

Input – Output flow Control: linear case



Input – Output flow Control: linear case

… but we also have delay effects!



$$\dot{y}(t) = -ay(t) + bu(t)$$

$$\dot{y}(t) = -ay(t) + bu(t-h)$$

$$\dot{y}(t) = -a_0 y(t) - a_1 y(t-\tau) + bu(t-h)$$

Simu Input Delay

Simu State Delay

Input – Output flow Control: linear case

… but we also have delay effects!

→ Translation in terms of Laplace transfers?

$$\frac{Y(s)}{U(s)} = \frac{b}{s+a}$$

$$\frac{Y(s)}{U(s)} = \frac{be^{-hs}}{s+a}$$

$$\frac{Y(s)}{U(s)} = \frac{be^{-hs}}{s+a_0+a_1e^{-\tau s}}$$

$$\dot{y}(t) = -ay(t) + bu(t)$$

$$\dot{y}(t) = -ay(t) + bu(t-h)$$

$$\dot{y}(t) = -a_0y(t) - a_1y(t-\tau) + bu(t-h)$$

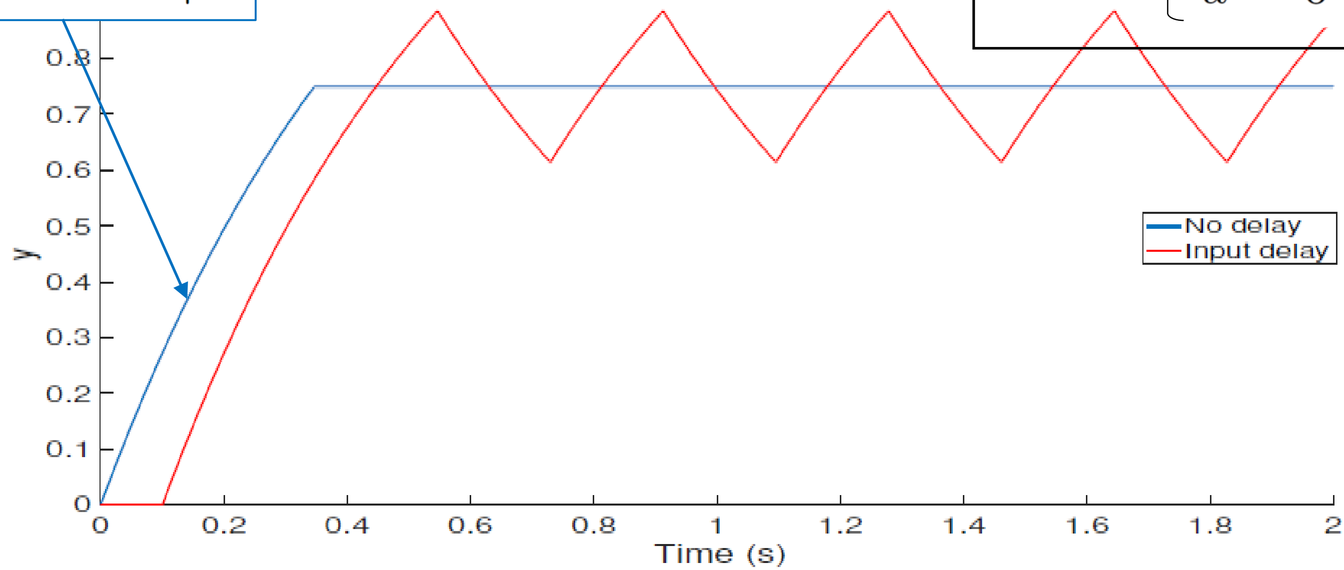
Input – Output flow Control: linear case

What is the effect of an input delay h on the previous sliding mode controller?

$$\begin{cases} u = 1 \text{ if } y(t) < y* \\ u = 0 \text{ if } y(t) > y* \end{cases}$$

$$\dot{y}(t) = -ay(t) + bu(t)$$

$$\frac{Y(s)}{U(s)} = \frac{b}{s+a}$$



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$$\frac{Y(s)}{U(s)} = \frac{be^{-hs}}{s+a}$$

Simu SMC Input Delay