Flow Control: TD4-1

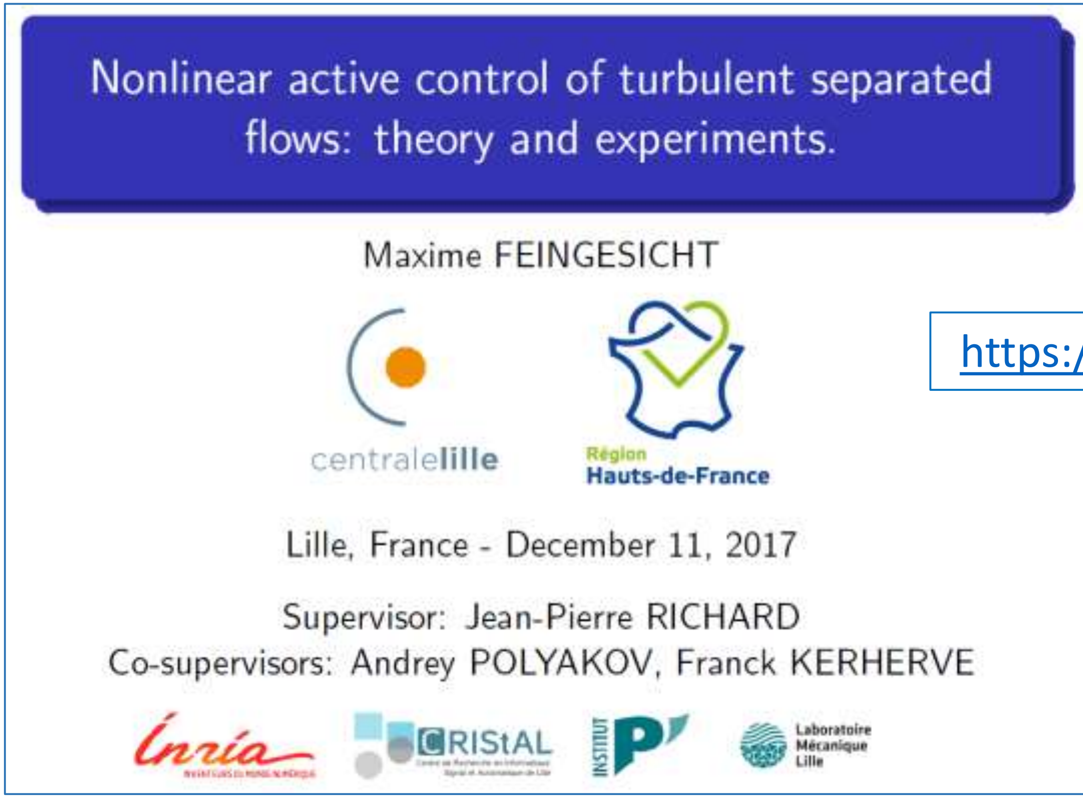
Overview

- General issues, passive *vs* active…
- Control issues: optimality and learning *vs* robustness and rough model
- Model-based control: linear model, nonlinear control
    - Linear model, identification
    - Sliding Mode Control
    - Delay effect
    - Time-delay systems
- Introduction to delay systems
    - Examples
    - Much a do about delay? Some special features + a bit of maths
    - Time-varying delay
- Model-based control: nonlinear model, nonlinear control
    - Overview of MF's PhD: Sliding Mode Control
    - Application to the airfoil
    - Application to the Ahmed body (MF and CC'PhDs)

…

- Machine Learning and model-free control: + 4h with Thomas Gomez

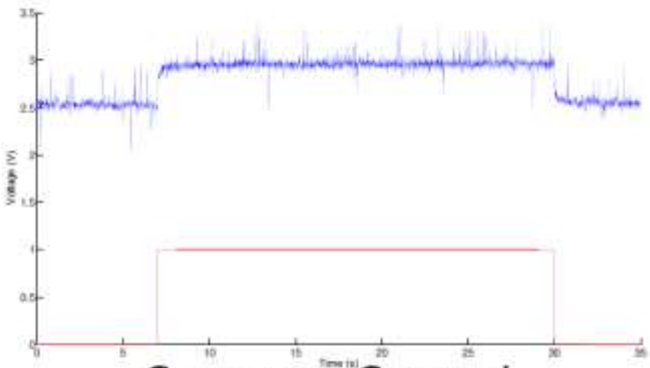
Flow Control



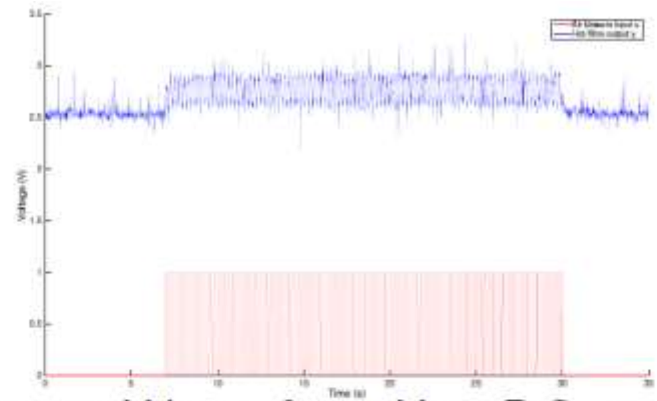
https://tel.archives-ouvertes.fr/tel-01801155

Back to the results from Maxime's PhD:
nonlinear delayed model with Sliding Mode Control

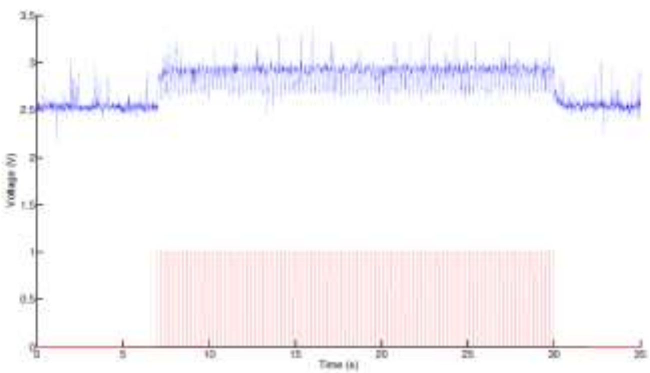
1) Model: identification



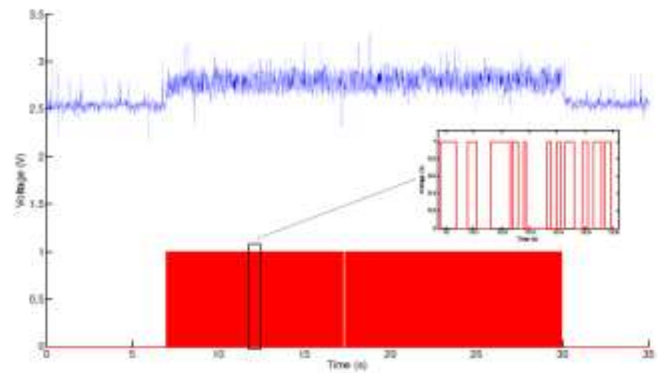
Constant Control



Square Wave, f = 4Hz , DC=50%



Square Wave, f = 4Hz , DC=80%



PRBS

$y(t) = y_c(t) - y_0(t)$, with $y_c(t)$ the current hot-film output and $y_0(t)$ the hot-film output with no control (approximately 2.5V).

1) Model: identification

Back to slide 36

Least-square identification [Ljung 1999] for fixed delays

$$
\left\{
\begin{array}{l}
W = \begin{bmatrix} y_{0-\tau_1} & \cdots & y_{0-\tau_{N_1}} & u_{0-h_1} & \cdots & u_{0-h_{N_3}} & y_{0-\tilde{\tau}_1} u_{0-h_1} & y_{0-\tilde{\tau}_2} u_{0-h_1} & \cdots & y_{0-\tilde{\tau}_{N_2}} u_{0-h_{N_3}} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ y_{(N-1)-\tau_1} & \cdots & y_{(N-1)-\tau_{N_1}} & u_{(N-1)-h_1} & \cdots & u_{(N-1)-h_{N_3}} & y_{(N-1)-\tilde{\tau}_1} u_{(N-1)-h_1} & y_{(N-1)-\tilde{\tau}_2} u_{(N-1)-h_1} & \cdots & y_{(N-1)-\tilde{\tau}_{N_2}} u_{(N-1)-h_{N_3}} \end{bmatrix} \\
A = \begin{bmatrix} a_0 & \dots & a_{N_1} & b & c_0 & \dots & c_{N_2} \end{bmatrix}^T \\
x = \begin{bmatrix} y_1 & \dots & y_N \end{bmatrix}^T \\
WA = x \Rightarrow A = W^+ x \; ; \; W^+ = \text{Moore-Penrose inverse of W}
\end{array}
\right. \quad (3)
$$

$N \sim 10^5$ to 3.10^6

Issue : $\min_{\tau,\tilde{\tau},h,A} \|y^{simu} - WA\|_2$ or $\max_{\tau,\tilde{\tau},h,A} \text{FIT}(y) \Rightarrow$ Necessity to use a nonlinear mixed integer programming algorithm, e.g. NOMAD ([Le Digabel 2011]) or Genetic Algorithm. The FIT coefficient is defined as (see [Dandois 2013]) :

$\text{FIT}(y) = 1 - \sqrt{\dfrac{\sum_{k=1}^{N} (y_k - y_k^{\text{simu}})^2}{\sum_{k=1}^{N} (y_k - \bar{y}_k)^2}}$, with $\bar{y}_k$ the average value of y up to the time step k and y_k^{simu} the output value of the model.

1) Model: identification

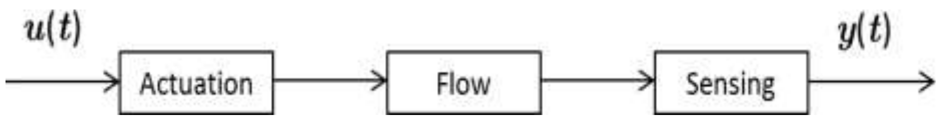
For a model with a FIT of approximately 80%, the coefficient vectors have the following sizes:

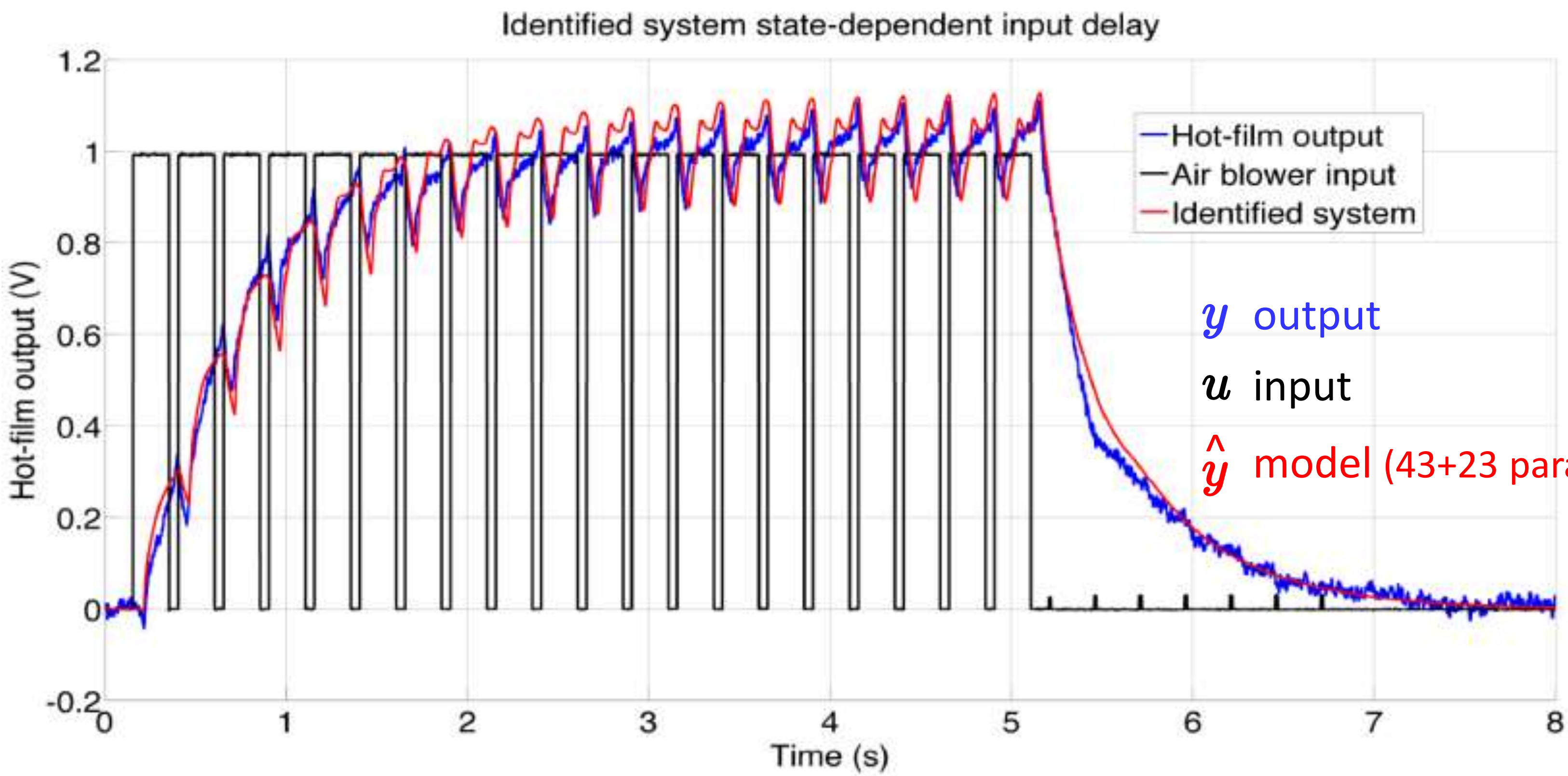
- τ and a : vectors of size 10
- $\tilde{\tau}$ and c : vectors of size 10
- h and b : vectors of size 3
- Total number of coefficients to find : 43 coefficients and 23 delays

Fluid Mechanics literature (with similar FIT) → 1000 to 10000 coefficients

NB: for our robust controller, 4 coefs + 3 delays = 7 !

1) Model: identification





y output

u input

$\hat{y}$ model (43+23 param.)

2) Control: a simplified model

4 coefs + 3 delays = 7 parameters

Continuous delayed bilinear model

$$\dot{y}(t) = a_1 y(t-h) - a_2 y(t-\tau) + \left(b - cy(t-h) + cy(t-\bar{\tau})\right) u(t-h) \quad (9)$$

Proposition 2: Positivity and boundedness of the system (Feingesicht et al. 2017b; Feingesicht et al. 2017c)

If $c < a_1$, $(a_1 + c)\tau < a_2\tau < \frac{1}{e}$ and $\tau \leq h \leq \bar{\tau}$
then the system (9) is positive and its solution is globally bounded for any input signal $u \in \mathbf{L}^\infty : u(t) \in \{0, 1\}$ as follows

$$0 \leq y(t) \leq y_{max} = \frac{b}{a_2 - a_1} \text{ for all } t \geq 0$$

2) Control: surface for the Sliding Mode Control

Sliding variable

We choose the following sliding variable:

$$\sigma(t) = y(t) - a_2 \int_{t-\tau}^{t} y(s)ds + c \int_{t-\bar{\tau}+h}^{t} y(s)ds$$

$$+ \int_{t-h}^{t} a_1 y(s) + (b - cy(s) + cy(s-\bar{\tau}+h))u(s)ds$$

Does that sound familiar?

Yes, predictor effect ☺

(slide 63)

It satisfies the equation:

$$\dot{\sigma}(t) = (a_1 - a_2 + c(1 - u(t)))y(t) + c(u(t) - 1)y(t - \bar{\tau} + h) + bu(t)$$

There is no delay in the control input u in the expression of the derivative of the sliding variable $\dot{\sigma}$. We use the sliding mode existence condition $\dot{\sigma}(t)\sigma(t) < 0$ from [Utkin, 1992] for control design.

2) Control: conditions for Sliding Mode

Proposition 3: Conditions for Sliding Mode (Feingesicht et al. 2017b; Feingesicht et al. 2017c)

If conditions of Proposition 2 hold and

$$Q(j\omega) \neq 0 \quad \text{for} \quad \omega \neq 0,$$

where $Q(s) = s + a_2 e^{-s\tau} - (a_2 - c)e^{-sh} - ce^{-s\bar{\tau}}, s \in \mathbb{C}$ *and* $j^2 = -1$, *then the control law*

$$u(t) = \begin{cases} 1 & \text{if} \quad \sigma(t) < \sigma^*, \\ 0 & \text{if} \quad \sigma(t) > \sigma^*, \end{cases}$$

with $\sigma^* = y^*(1 + a_2(h - \tau) + c(\bar{\tau} - h))$ *and* $y^* \in \left(0, \frac{b}{a_2 - a_1}\right)$ *guarantees* $y(t) \to y^*$ *as* $t \to +\infty$.

A more general version of this proposition for any number of delays is presented in the patent: M. Feingesicht, A. Polyakov, F. Kerherve and J. P. Richard, **Contrôle par modes glissants du décollement d'un écoulement**, *FR1755440, deposited 15/06/2017*

2) Control: proof of robustness w.r.t. additive perturbation

Corollary 1: Robustness

Let the model (9) contain an additive perturbation:

$$\dot{y}(t) = a_1 y(t-h) - a_2 y(t-\tau) + \left(b - cy(t-h) + cy(t-\bar{\tau})\right) u(t-h) + d(t) \quad (10)$$

where $d(t)$ is a bounded signal $\|d(t)\| < \bar{d}$ such that (10) remains a positive system. If

$$\bar{d} < \frac{b}{1 + \frac{a_2 - a_1}{a_2 - a_1 - c}}$$

then $y(t) \to y^* + O(h\bar{d})$ as $t \to +\infty$ for $y^* \in \left(\frac{\bar{d}}{a_2 - a_1 - c}, \frac{b - \bar{d}}{a_2 - a_1}\right)$.

Conjecture for future developments

The condition

$$Q(j\omega) \neq 0 \quad \text{for} \quad \omega \neq 0,$$

may not be necessary and can be removed from Proposition 3.

2) Control: summary

$$\sigma(t) = y(t) - a_2 \int_{t-\tau}^{t} y(s)ds + c \int_{t-\bar{\tau}+h}^{t} y(s)ds + \int_{t-h}^{t} a_1 y(s) + (b - cy(s) + cy(s-\bar{\tau}+h))u(s)ds$$

Proposition 2 : Conditions for Sliding Mode

If conditions of Proposition 1 hold and

$$Q(j\omega) \neq 0 \quad \text{for} \quad \omega \neq 0, \tag{7}$$

where $Q(s) = s + a_2 e^{-s\tau} - (a_2 - c)e^{-sh} - ce^{-s\bar{\tau}}$, $s \in \mathbb{C}$ *and* $j^2 = -1$, *then the control law*

$$u(t) = \begin{cases} 1 & \text{if} \quad \sigma(t) < \sigma^*, \\ 0 & \text{if} \quad \sigma(t) > \sigma^*, \end{cases} \tag{8}$$

with $\sigma^* = y^*(1 + a_2(h - \tau) + c(\bar{\tau} - h))$ *and* $y^* \in \left(0, \frac{b}{a_2 - a_1}\right)$ *guarantees* $y(t) \to y^*$ *as* $t \to +\infty$.

2) Control: check on simulation

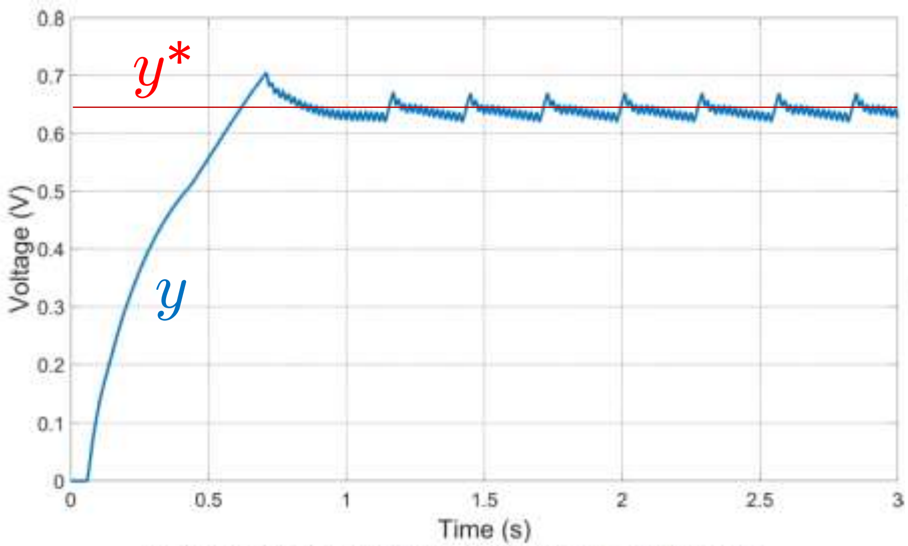


Figure 6. Application of the setpoint tracking control: Output of the system

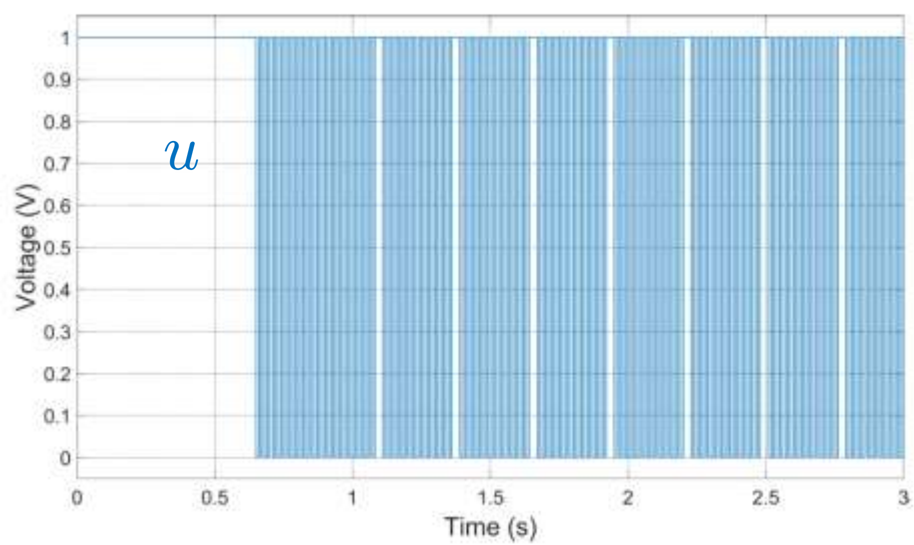


Figure 7. Application of the setpoint tracking control: Control signal

Jan. 2015:	PhD starts
March 2016:	control ok on simulations ☺
October:	journal paper submitted ☺
December:	still looking for platform's availability… ☹
Feb. 2017:	actuated platform ready!

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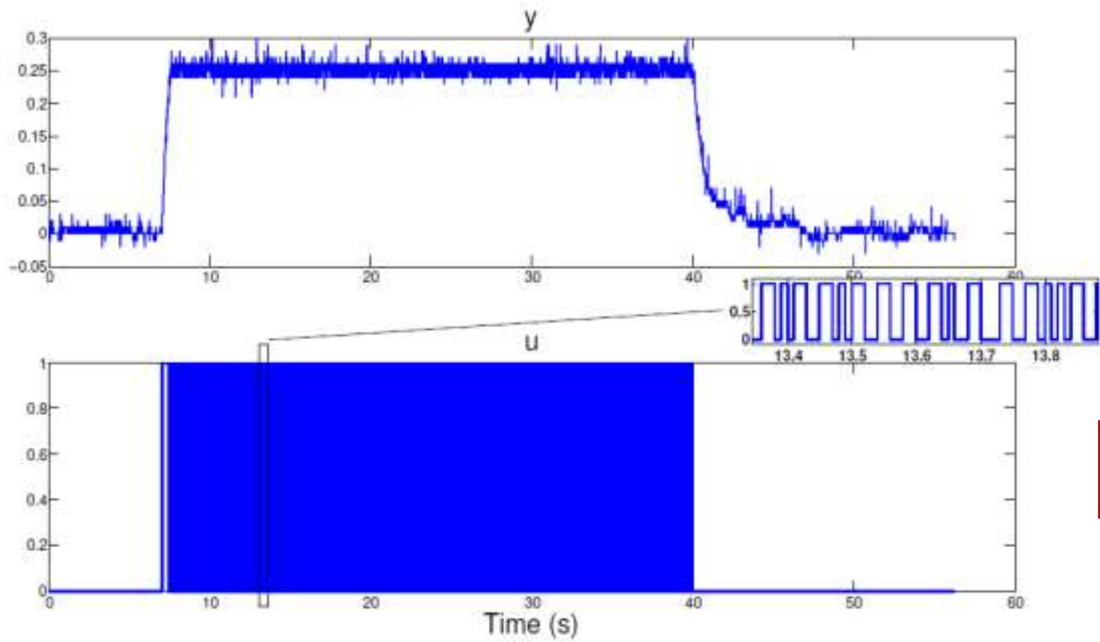
3) Experiments!

$v = 34.5m/s$; $AoA = 24°$; $q = 25g/s$; $y^* = 0.25$

The model was identified for a flow speed $v = 34.5m/s$, an angle of attack $AoA = 24°$ and an actuator mass flow $q = 25g/s$ (nominal conditions).



nominal

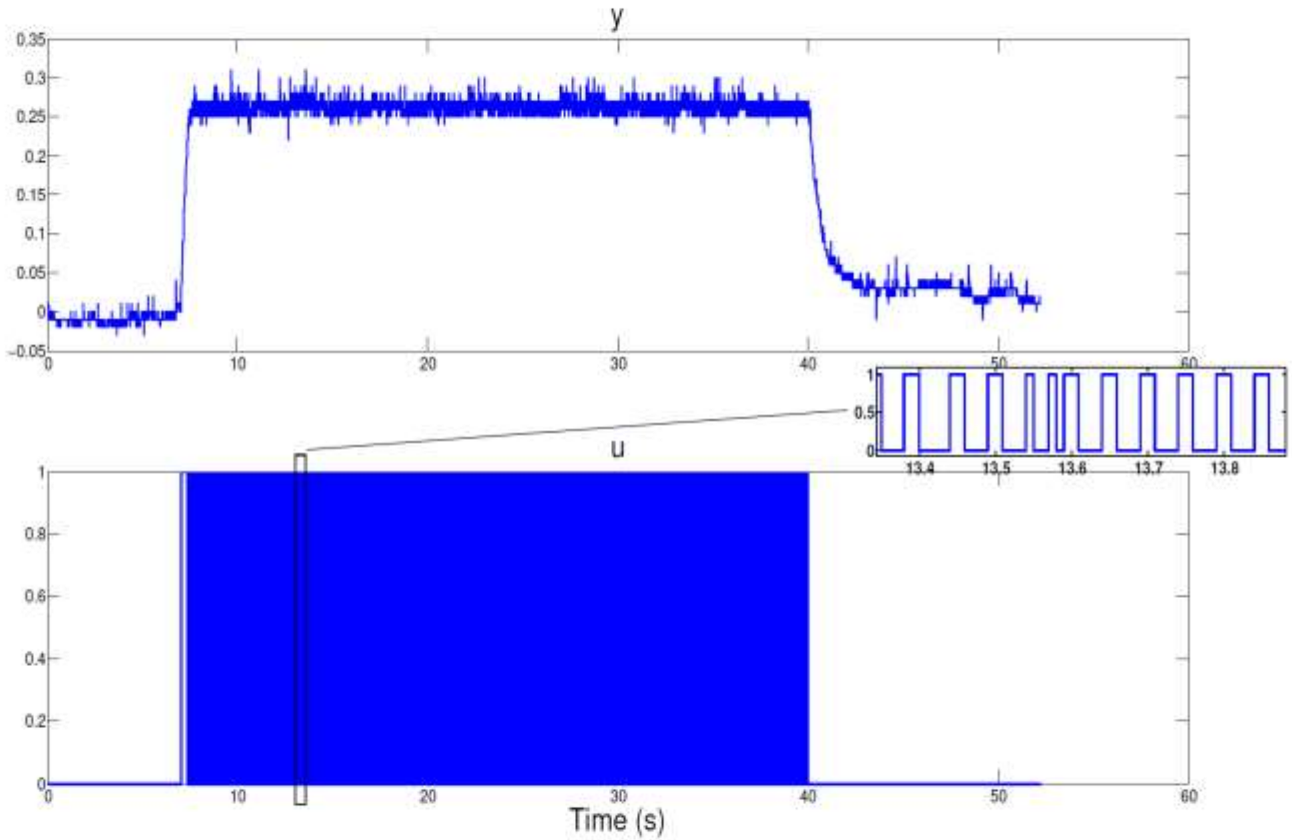
3) Experiments

https://www.youtube.com/watch?v=b5NnAV2qeno

$v = 14.5m/s$; $AoA = 24^\circ$; $q = 25g/s$; $y^* = 0.25$

v	flow speed
AoA	angle of the flap
q	mass flow of the actuators

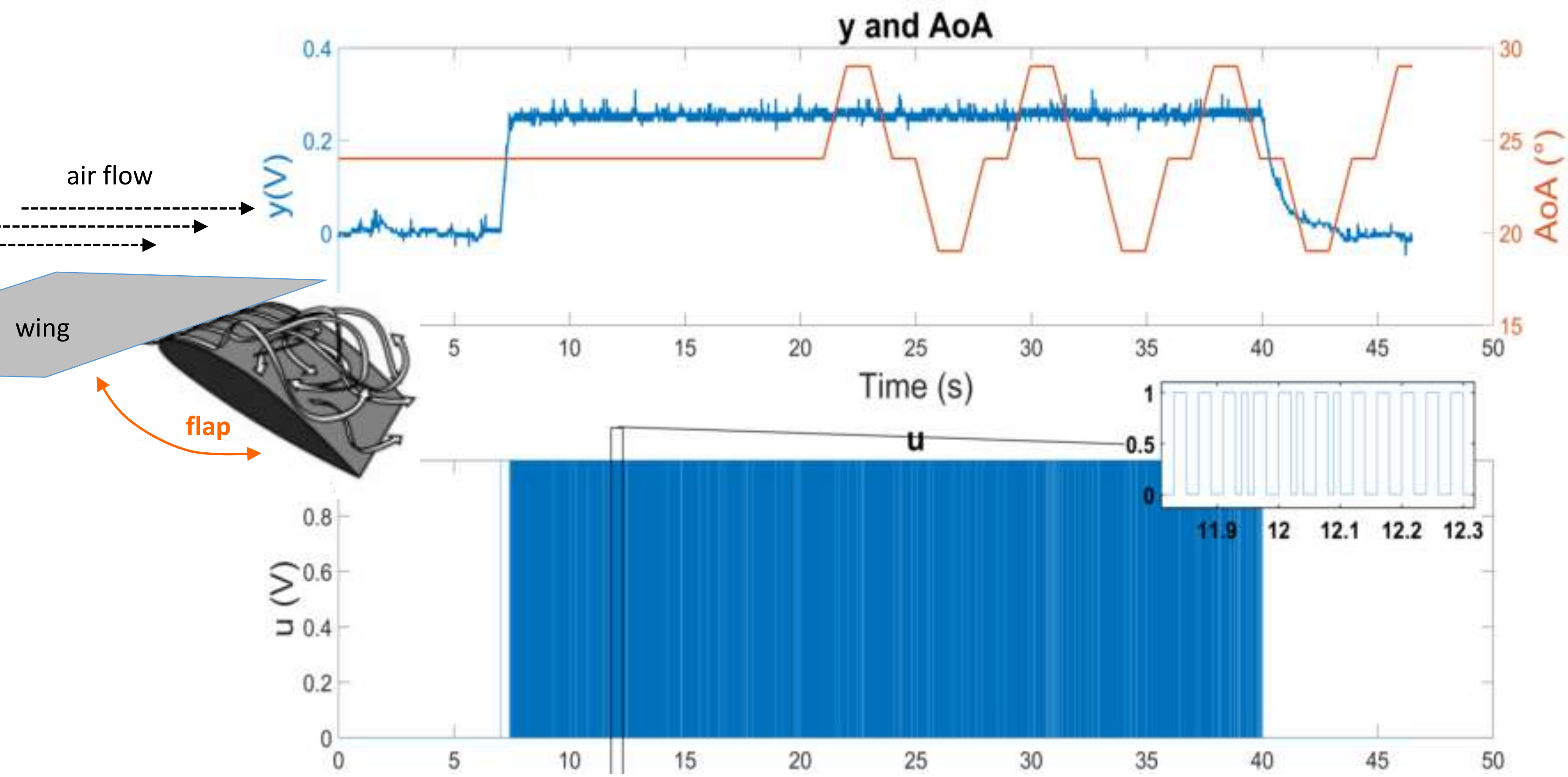
Non nominal: lower flow speed



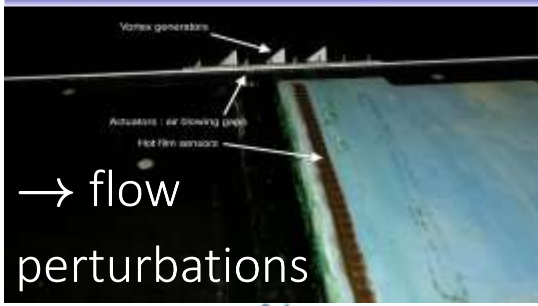
$v = 34.5m/s$; AoA = Time varying ; $q = 25g/s$; $y^* = 0.25$

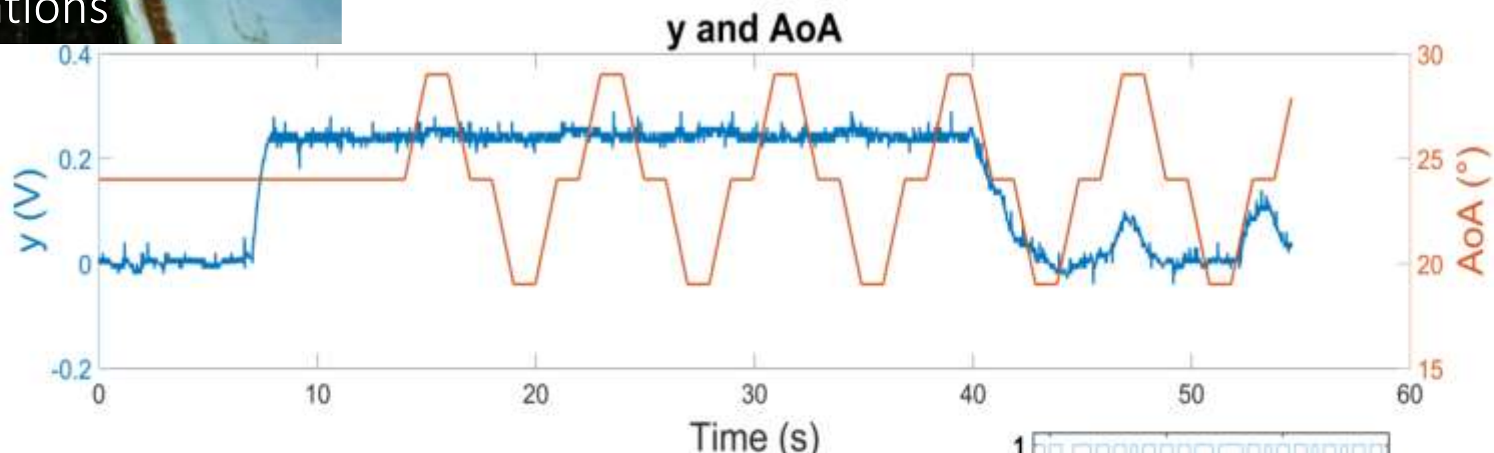
v	flow speed
AoA	angle of the flap
q	mass flow of the actuators

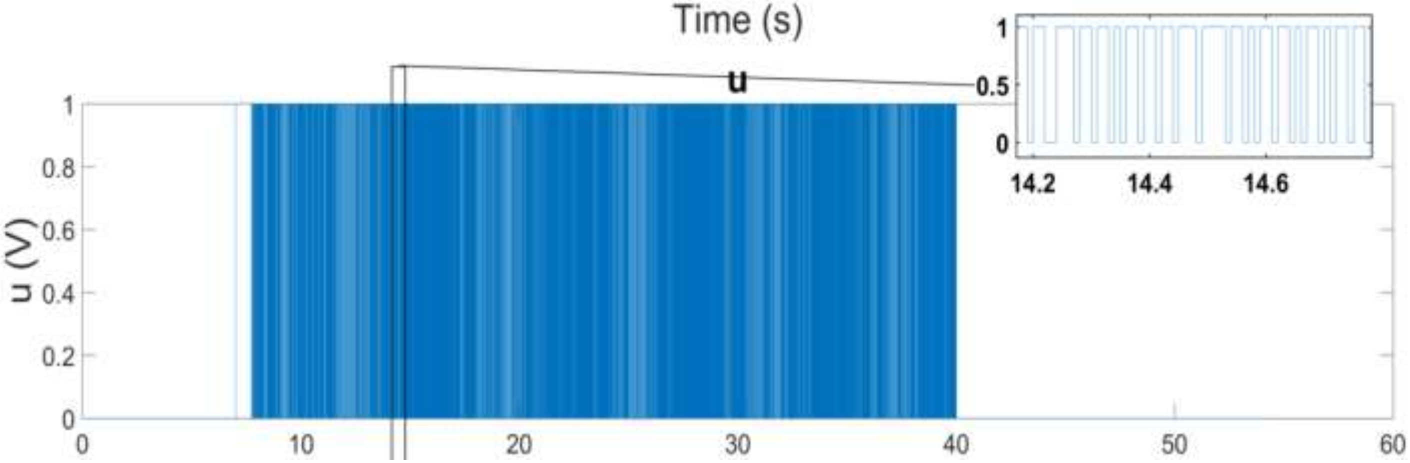
flap angle variations
AoA = 20°to 38°



$v = 34.5m/s$; AoA = Time varying ; $q = 25g/s$; $y^* = 0.25$; vortex generators added before the flap



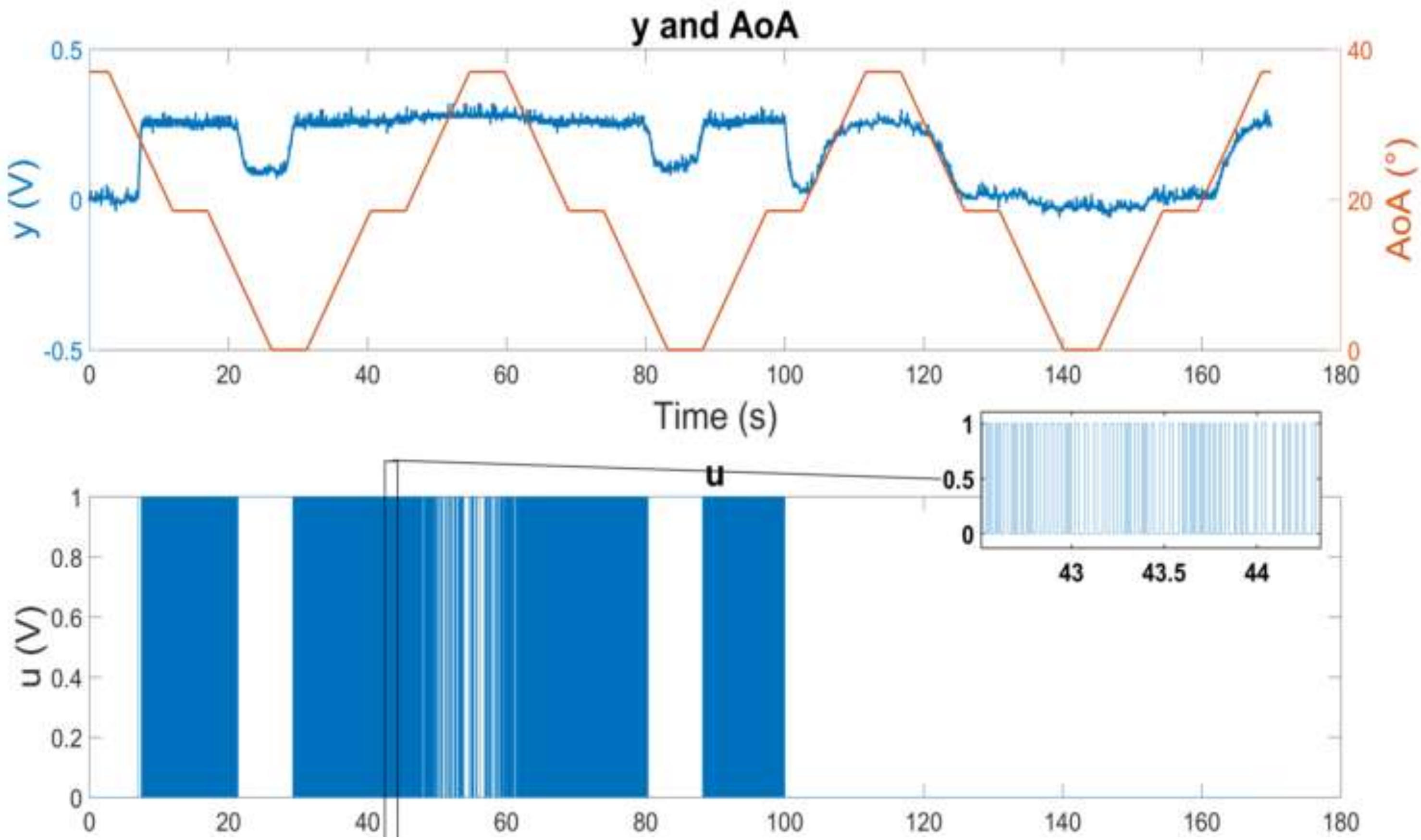




$v = 34.5m/s$; AoA = Time varying ; $q = 25g/s$; $y^* = 0.25$

v	flow speed
AoA	angle of the flap
q	mass flow of the actuators

largest angle variations
AoA = 0°to 38°



Flow Control: conclusions

- Simple models were developed for the studied flows
- These models come from intuition and have been shown to be an approximation of the Burgers equation
- The models can reach the same accuracy as the ones in the literature with 100 to 1000 times less coefficients
- Sliding Mode Control is perfectly adapted to the system and gives very good results with experimentally tested robustness
- The control was implemented using an Arduino, therefore it is not computationally or memory expensive

Flow Control: perspectives

Theoretical perspectives:

- Extension of the model and control to MIMO models
- Study the effect of discretization on the Sliding Mode control
- Use frequency analysis on the control signals and compare it to natural frequencies of the flow

Practical perspectives:

- Application of the control in the MIMO case to the Ahmed body in LAMIH with measurement of drag and lift and 3 actuators
- Application on a real car with LAMIH-UVHC. The car is equipped with pressures sensors to estimate the drag and air jets actuators and is tested on French highway
- Application on flexible airplane (Michigan University), trains (Faiveley Transport) and drones (InterReg European project submitted with U. Southampton, TU Delft, UVHC, ONERA, Inria, Alstom, i-Trans, Railenium)