Sliding mode control of systems with time-varying delays via descriptor approach

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Abstract

In this paper, we combine a descriptor approach to stability and control of linear systems with time-varying delays, which is based on the Lyapunov - Krasovskii techniques, with a recent result on sliding mode control of such systems. The systems under consideration have norm-bounded uncertainties and uncertain bounded delays. The solution is given in terms of linear matrix inequalities (LMIs) and improves the previous results based on other Lyapunov techniques. A numerical example illustrates the advantages of the new method.

1 Introduction

The interest in robust control of time-delay systems this last decade is witnessed by the rich dedicated literature (see for instance, [1]- [17] and the numerous references therein). Many existing results concern systems with unknown but constant delays. But in some applications, such as networked control or tele-operated systems, the assumption of a constant delay is too restrictive: this can lead to bad performances or, even worse, to unstability

This paper combines two previous results so to obtain a more efficient sliding mode controller for uncertain systems with time-varying delays and norm-bounded uncertainties. Other results [9] concern varying delays but may lead to strong conditions which reduces the dynamic performances.

The first of these results is the sliding mode design given in [9], which copes with stabilization of systems with time-varying delays. The approach relies on the construction on a Lyapunov-Razumikhin function which allows fast variations of the delay but leads to some conservatism on the upper bound of the time-delay.

The second result given in [3] concerns the construction of a new class of Lyapunov-Krasovskii functionals using a descriptor model transformation. Unlike previous transformations, the descriptor model leads to a system which is equivalent to the original one (from the point of view of stability) and requires bounding of fewer cross-terms. Furthermore, following this approach, stability criteria have been given in [6] for systems with time-varying delays without any assumption on their derivatives (which was the case with the usual Lyapunov-Krasovskii functionals).

The paper is organized as follows: In section 2, we develop a Lyapunov-Krasovskii approach on a descriptor representation for an uncertain, linear, time-delay system. This provides a stability condition expressed in term of feasibility of a linear matrix inequality (LMI) (see [1]). Then, the design of a stabilizing memoryless state feedback is derived. Section 3 deals with the design of a sliding mode controller. This is achieved through the resolution of a generalized eigenvalue problem which can be solved efficiently using semi-definite programming tools. In the last section, an illustrative example is solved using our approach and comparison with previous results are provided.

Notation:

Throughout the paper the superscript 'T' stands for matrix transposition, $\mathcal{R}^n$ denotes the n dimensional Euclidean space, $\mathcal{R}^{n\times m}$ is the set of all $n\times m$ real matrices. The notation $P>0$, for $P\in\mathcal{R}^{n\times n}$ means that P is symmetric and positive definite. I_n represents the $n\times n$ identity matrix.

2 Stabilization of linear systems with norm-bounded uncertainties by delayed feedback

In this section we consider the following uncertain linear system with a time-varying delay:

$$\begin{aligned}\dot{x}(t) &= (A_0+H\Delta(t)E_0)x(t)+(A_1+H\Delta(t)E_1)x(t-\tau(t))+(B_0+H\Delta(t)E_2)u(t)+B_1u(t-\tau(t)),\\ x(t)&=\phi(t),\ t\in[-h,0],\end{aligned}\tag{1}$$

where $x(t)\in\mathcal{R}^n$ is the system state, $u(t)\in\mathcal{R}^m$ is the control input, h is an upper-bound on the time-delay function ($0\le\tau(t)\le h$, $\forall t\ge 0$). The matrix $\Delta(t)\in\mathcal{R}^{p\times q}$ is a matrix of time-varying, uncertain parameters satisfying

$$\Delta^T(t)\Delta(t)\le I_q\ \ \forall\, t.\tag{2}$$

For simplicity, we consider only one delay, but the results of this section may be easily generalized to the case of multiple delays.

We seek a control law

$$u(t)=Kx(t)\tag{3}$$

that will asymptotically stabilize the system.

2.1 The stability issue

In this subsection, we consider the following equation:

$$\dot{x}(t) = (\bar{A}_0 + H\Delta(t)\bar{E}_0)x(t) + (\bar{A}_1 + H\Delta(t)\bar{E}_1)x(t-\tau(t)). \tag{4}$$

Representing (1) in an equivalent descriptor form [3]:

$$\dot{x}(t) = y(t), \quad 0 = -y(t) + (\bar{A}_T + H\Delta\bar{E}_T)x(t) - (\bar{A}_1 + H\Delta\bar{E}_1)\int_{t-\tau(t)}^{t} y(s)ds$$

or

$$E\dot{\bar{x}}(t) = \begin{bmatrix} 0 & I_n \\ \bar{A}_T + H\Delta\bar{E}_T & -I_n \end{bmatrix}\bar{x}(t) - \begin{bmatrix} 0 \\ \bar{A}_1 + H\Delta\bar{E}_1 \end{bmatrix}\int_{t-\tau(t)}^{t} y(s)ds, \tag{5}$$

with

$$\begin{aligned} \bar{x}(t) &= col\{x(t),\ y(t)\},\ E = diag\{I_n, 0\}, \\ \bar{A}_T &= \bar{A}_0 + \bar{A}_1, \bar{E}_T = \bar{E}_0 + \bar{E}_1, \end{aligned}$$

the following Lyapunov-Krasovskii functional is applied:

$$V(t) = \bar{x}^T(t)EP\bar{x}(t) + V_2(t), \tag{6}$$

where

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \quad P_1 > 0, \quad EP = P^TE \geq 0, \tag{7a-d}$$

$$V_2(t) = \int_{-h}^{0}\int_{t+\theta}^{t} y^T(s)[R + \delta_2\bar{E}_1^T\bar{E}_1]y(s)dsd\theta.$$

The following result is obtained:

Lemma 1 *The system (4) is asymptotically stable if there exist* $n\times n$ *matrices* $0<P_1,\ P_2,\ P_3,\ R>0$ *and positive numbers* δ_1, δ_2 *that satisfy the following LMI:*

$$\Gamma = \begin{bmatrix} \Psi & hP^T\begin{bmatrix} 0 \\ \bar{A}_1 \end{bmatrix} & P^T\begin{bmatrix} 0 \\ H \end{bmatrix} & hP^T\begin{bmatrix} 0 \\ H \end{bmatrix} \\ * & -hR & 0 & 0 \\ * & * & -\delta_1 I_p & 0 \\ * & * & * & -\delta_2 h I_p \end{bmatrix} < 0 \tag{8}$$

where

$$\Psi = \Psi_0 + \begin{bmatrix} \delta_1\bar{E}_T^T\bar{E}_T & 0 \\ 0 & h(R + \delta_2\bar{E}_1^T\bar{E}_1) \end{bmatrix},$$

$$\Psi_0 = P^T\begin{bmatrix} 0 & I_n \\ \bar{A}_T & -I_n \end{bmatrix} + \begin{bmatrix} 0 & I_n \\ \bar{A}_T & -I_n \end{bmatrix}^T P,$$

and $*$ *denotes symmetrical entries.*

Proof. Note that

$$\bar{x}^T(t)EP\bar{x}(t) = x^T(t)P_1x(t)$$

and, hence, differentiating the first term of (6) with respect to t gives:

$$\frac{d}{dt}\{\bar{x}^T(t)EP\bar{x}(t)\} = 2x^T(t)P_1\dot{x}(t) = 2\bar{x}^T(t)P^T \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix}. \tag{9}$$

Replacing $\begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix}$ by the right side of (5) we obtain:

$$\frac{dV(t)}{dt} = \bar{x}^T(t)\Psi_0\bar{x}(t) + \eta_0 + \eta_1 + \eta_2 + hy^T(t)[R + \delta_2\bar{E}_1^T\bar{E}_1]y(t) - \int_{t-h}^{t} y^T(s)[R + \delta_2\bar{E}_1^T\bar{E}_1]y(s)ds, \tag{10}$$

where

$$\eta_0(t) \triangleq -2\int_{t-\tau(t)}^{t} \bar{x}^T(t)P^T \begin{bmatrix} 0 \\ \bar{A}_1 \end{bmatrix} y(s)ds,$$

$$\eta_1(t) \triangleq 2\bar{x}^T(t)P^T \begin{bmatrix} 0 \\ H \end{bmatrix} \Delta(\bar{E}_0 + \bar{E}_1)x(t),$$

$$\eta_2(t) \triangleq -2\int_{t-\tau(t)}^{t} \bar{x}^T(t)P^T \begin{bmatrix} 0 \\ H \end{bmatrix} \Delta\bar{E}_1 y(s)ds.$$

Applying the standard bounding

$$a^Tb \leq a^TRa + b^TR^{-1}b, \quad \forall a, b \in \mathcal{R}^n, \forall R \in \mathcal{R}^{n\times n} : R > 0,$$

and using the fact that $\tau(t) \leq h$, we have

$$\begin{aligned} \eta_0(t) &\leq \tau\bar{x}^T(t)P^T \begin{bmatrix} 0 \\ \bar{A}_1 \end{bmatrix} R^{-1}[0\ \bar{A}_1^T]P\bar{x}(t) + \textstyle\int_{t-\tau(t)}^{t} y^T(s)Ry(s)ds \\ &\leq h\bar{x}^T(t)P^T \begin{bmatrix} 0 \\ \bar{A}_1 \end{bmatrix} R^{-1}[0\ \bar{A}_1^T]P\bar{x}(t) + \textstyle\int_{t-h}^{t} y^T(s)Ry(s)ds. \end{aligned} \tag{11}$$

Similarly

$$\eta_1 \leq \delta_1^{-1}\bar{x}^T(t)P^T \begin{bmatrix} 0 \\ H \end{bmatrix} [0\ H^T]P\bar{x}(t) + \delta_1 x^T(t)\bar{E}_T^T\bar{E}_T x(t),$$

$$\eta_2 \leq h\delta_2^{-1}\bar{x}^T(t)P^T \begin{bmatrix} 0 \\ H \end{bmatrix} [0\ H^T]P\bar{x}(t) + \delta_2\int_{t-h}^{t} y^T(s)\bar{E}_1^T\bar{E}_1 y(s)ds.$$

Substituting the right sides of the latter inequalities into (10), we obtain

$$\frac{dV(t)}{dt} \leq \bar{x}^T(t)\bar{\Gamma}\bar{x}(t) \tag{12}$$

where

$$\bar{\Gamma} = \Psi + hP^T \begin{bmatrix} 0 \\ \bar{A}_1 \end{bmatrix} R^{-1}[0\ \bar{A}_1^T]P + (\delta_1^{-1} + h\delta_2^{-1})P^T \begin{bmatrix} 0 \\ H \end{bmatrix} [0\ H^T]P.$$

Therefore, LMI (8) yields by Schur complements that $\bar{\Gamma} < 0$ and hence $\dot{V} < 0$, while $V \geq 0$, and thus (4) is asymptotically stable [13], [4]. ♣

2.2 State-feedback stabilization

The results of Lemma 1 can also be used to verify the stability of the closed-loop obtained by applying (3) to the system (1) if we set in (8)

$$\bar{A}_i = A_i + B_i K,\ i = 0, 1, \quad \bar{E}_0 = E_0 + E_2 K \tag{13}$$

and verify that the resulting LMI is feasible. The problem with (8) is that it is linear in its variables only when the state-feedback gain K is given. In order to find K we apply again Schur formula to $\bar{\Gamma}$, the Ψ term being expanded. We thus obtain the following matrix inequality:

$$\begin{bmatrix} \Psi_0 & hP^T \begin{bmatrix} 0 \\ \bar{A}_1 R^{-1} \end{bmatrix} & \begin{bmatrix} 0 \\ hI_n \end{bmatrix} & \begin{bmatrix} \bar{E}_T^T \\ 0 \end{bmatrix} & h \begin{bmatrix} 0 \\ \bar{E}_1^T \end{bmatrix} & \delta_1^{-1} P^T \begin{bmatrix} 0 \\ H \end{bmatrix} & \delta_2^{-1} hP^T \begin{bmatrix} 0 \\ H \end{bmatrix} \\ * & -hR^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & -hR^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & -\delta_1^{-1} I_q & 0 & 0 & 0 \\ * & * & * & * & -\delta_2^{-1} hI_q & 0 & 0 \\ * & * & * & * & * & -\delta_1^{-1} I_p & 0 \\ * & * & * & * & * & * & -\delta_2^{-1} hI_p \end{bmatrix} < 0 \tag{14}$$

Consider the inverse of P. It is obvious, from the requirement $P_1 > 0$ and the fact that in (8) $-(P_3 + P_3^T)$ must be negative definite, that P is nonsingular. Defining

$$P^{-1} = Q = \begin{bmatrix} Q_1 & 0 \\ Q_2 & Q_3 \end{bmatrix} \quad \text{and} \quad M = diag\{Q,\ I_{2(n+p+q)}\} \tag{15a-b}$$

we multiply (14) by M^T and M, on the left and on the right, respectively. Choosing

$$R^{-1} = Q_1 \varepsilon,$$

where ε is a positive number, and introducing $\bar{\delta}_1 = \delta_1^{-1}$ and $\bar{\delta}_2 = \delta_2^{-1}$, we obtain the LMI

$$\left[\begin{array}{ccccccc} \Phi & h\begin{bmatrix} 0 \\ \bar{A}_1 Q_1 \varepsilon \end{bmatrix} & Q^T \begin{bmatrix} 0 \\ hI \end{bmatrix} & Q^T \begin{bmatrix} \bar{E}_T^T \\ 0 \end{bmatrix} & hQ^T \begin{bmatrix} 0 \\ \bar{E}_1^T \end{bmatrix} & \bar{\delta}_1 \begin{bmatrix} 0 \\ H \end{bmatrix} & h\bar{\delta}_2 \begin{bmatrix} 0 \\ H \end{bmatrix} \\ * & -hQ_1\varepsilon & 0 & 0 & 0 & 0 & 0 \\ * & * & -hQ_1\varepsilon & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{\delta}_1 I_q & 0 & 0 & 0 \\ * & * & * & * & -h\bar{\delta}_2 I_q & 0 & 0 \\ * & * & * & * & * & -\bar{\delta}_1 I_p & 0 \\ * & * & * & * & * & * & -\bar{\delta}_2 h I_p \end{array}\right] < 0 \qquad (16)$$

where

$$\Phi = \begin{bmatrix} 0 & I_n \\ \bar{A}_T & -I_n \end{bmatrix} Q + Q^T \begin{bmatrix} 0 & I_n \\ \bar{A}_T & -I_n \end{bmatrix}^T.$$

Substituting (13) into (16) and denoting $Y = KQ_1$, $B_T = B_0 + B_1$, we obtain

Theorem 1 *The control law of (3) asymptotically stabilizes (1) if, for some positive number ε, there exist scalars $\bar{\delta}_1 > 0, \bar{\delta}_2 > 0$ and matrices $0 < Q_1,\ Q_2,\ Q_3, \in \mathcal{R}^{n\times n}$ $Y \in \mathcal{R}^{m\times n}$ that satisfy the following LMI:*

$$\left[\begin{array}{cccc} Q_2 + Q_2^T & Q_1 A_T^T + Y^T B_T^T - Q_2^T + Q_3 & 0 & hQ_2^T \\ * & -Q_3 - Q_3^T & h\varepsilon(A_1 Q_1 + B_1 Y) & hQ_3^T \\ * & * & -h\varepsilon Q_1 & 0 \\ * & * & * & -hQ_1\varepsilon \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{array}\right. \qquad (17)$$

$$\begin{bmatrix} Q_1E_T^T + Y^TE_2^T & hQ_2^TE_1^T & 0 & 0 \\ 0 & hQ_3^TE_1^T & \bar{\delta}_1 H & h\bar{\delta}_2 H \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\bar{\delta}_1 I_q & 0 & 0 & 0 \\ * & -h\bar{\delta}_2 I_q & 0 & 0 \\ * & * & -\bar{\delta}_1 I_p & 0 \\ * & * & * & -\bar{\delta}_2 h I_p \end{bmatrix} < 0 \tag{18}$$

The state-feedback gain is then given by

$$K = YQ_1^{-1}. \tag{19}$$

3 Sliding mode controller

In this section, we focus on time-delay systems that can be represented, possibly, after a change of state coordinates and input, in the following regular form ([9],[18]):

$$\begin{cases} \dfrac{dz_1(t)}{dt} = & (A_{11} + H\Delta(t)E_0)z_1(t) + (A_{d11} + H\Delta(t)E_1)z_1(t-\tau(t)) \\ & +(A_{12} + H\Delta(t)E_2)z_2(t) + A_{d12}z_2(t-\tau(t)) \\ \dfrac{dz_2(t)}{dt} = & \sum_{i=1}^{2}(A_{2i}z_i(t) + A_{d2i}z_i(t-\tau)) + Du(t) + f(t,z_t), \\ z(t) = \phi(t) \text{ for } t \in [-h,0] \end{cases} \tag{20}$$

where $z(t) = (z_1, z_2)^T$, $z_1 \in \mathcal{R}^{n-m}$, $z_2 \in \mathcal{R}^m$, $A_{ij}, A_{dij}, i = 1,2,\ j = 1,2,\ E_k,\ k = 0,1,2, H$ are constant matrices of appropriate dimensions, D is a regular $m \times m$ matrix, the matrix $\Delta(t)$ is a time-varying matrix of uncertain parameters, $u \in \mathcal{R}^m$ is the input vector, τ is time-varying delay satisfying $0 \le \tau(t) \le h$, $\forall t \ge 0$, $z_t(\theta)$ is the function associated with z and defined on $[-h, 0]$ by $z_t(\theta) = z(t+\theta)$, ϕ is the initial piecewise continuous function defined on $[-h, 0]$.

We will assume that:

A1) $(A_{11} + A_{d11}, A_{12} + A_{d12})$ is controllable.

A2) f is Lipschitz continuous and satisfies the inequality

$$\|f(t,z_t)\| < F_M(t,z_t), \quad \forall t \ge 0,$$

where $F_M(t,z_t)$ is a continuous functional assumed to be known *a priori*,

A3) $\Delta(t)$ is a time-varying matrix of uncertain parameters satisfying $\Delta^T(t)\Delta(t) \le I\ \ \forall\, t$.

Consider the following switching function:

$$s(z) = z_2 - Kz_1 \tag{21}$$

with $K \in \mathcal{R}^{m\times(n-m)}$. Let Ω, Θ be the linear functions defined by

$$\begin{array}{c} \Omega(z(t)) = \sum_{i=1}^{2}(A_{2i} - KA_{1i})z_i(t), \\ \Theta(z(t)) = E_0 z_1(t) + E_2 z_2(t) \end{array} \tag{22}$$

and let D_M be the following functional:

$$\begin{aligned} D_M(z_t) \;&=\; (\|A_{d21} - KA_{d11}\| + \|KH\|\,\|E_1\|) \sup_{-h\le\theta\le 0} \|z_1(t+\theta)\| \\ &\quad + \|A_{d22} - KA_{d12}\| \sup_{-h\le\theta\le 0} \|z_2(t+\theta)\|. \end{aligned} \tag{23}$$

Following [9] and using the results of previous section, we are able to design a sliding mode controller that will stabilize system (20) under less conservative assumptions on the delay law.

Theorem 2 *Assume A1-A3. If, for some positive number* ε*, there exist positive numbers* $\bar{\delta}_1, \bar{\delta}_2$ *and matrices* $0 < Q_1,\ Q_2,\ Q_3 \in \mathcal{R}^{(n-m)\times(n-m)},\ Y \in \mathcal{R}^{m\times(n-m)}$ *that satisfy the following LMI:*

$$\left[\begin{array}{cccccccc} Q_2 + Q_2^T & X_{12} & 0 & hQ_2^T & Q_1E_T^T + Y^TE_2^T & hQ_2^TE_1^T & 0 & 0 \\ * & -Q_3 - Q_3^T & h\varepsilon(A_{d11}Q_1 + A_{d12}Y) & hQ_3^T & 0 & hQ_3^TE_1^T & \bar{\delta}_1 H & h\bar{\delta}_2 H \\ * & * & -h\varepsilon Q_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -h\varepsilon Q_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\bar{\delta}_1 I & 0 & 0 & 0 \\ * & * & * & * & * & -h\bar{\delta}_2 I & 0 & 0 \\ * & * & * & * & * & * & -\bar{\delta}_1 I & 0 \\ * & * & * & * & * & * & * & -\bar{\delta}_2 hI \end{array}\right] < 0. \tag{24}$$

where

$$X_{12} = Q_1(A_{11}^T + A_{d11}^T) + Y^T(A_{12}^T + A_{d12}^T) - Q_2^T + Q_3,$$

then the sliding mode control law

$$u(t) = -D^{-1}\left[\Omega(z(t)) + (F_M(t,z_t) + D_M(z_t) + \|KH\|\,\|\Theta(z(t))\| + M)\frac{s(z(t))}{\|s(z(t))\|}\right], \tag{25}$$

where $K = YQ_1^{-1},\ M > 0$ *and* s, Ω, Θ, D_M *are defined in (21)-(23) , asymptotically stabilizes system (20) for any delay function* $\tau(t) \le h$.

Proof : The proof is divided into two parts. The first one is dedicated to the proof of the existence of an ideal sliding motion on the surface $s(z) = 0$, the second part to the proof of the stability of the reduced system.

Attractivity of the manifold:

Consider the Lyapunov-Krasovskii functional

$$V(t) = s^T(z(t))s(z(t)) = \|s(z(t))\|^2 . \quad (26)$$

Differentiating (26) on the trajectories of the closed-loop system gives

$$\begin{aligned}\dot{V}(t) &= 2s^T(t)(\Omega(z(t)) + \sum_{i=1}^{2}[A_{d2i} - KA_{d1i}]\, z_i(t-\tau) + Du(t) + \\ &\quad f(t, z_t) - KH\Delta(t)[\Theta(z(t)) + E_1 z_1(t-\tau(t))]),\end{aligned}$$

Using the expression of the control law (25), we get

$$\begin{aligned}\dot{V}(t) &= 2s^T(t)(\sum_{i=1}^{2}(A_{d2i} - KA_{d1i})z_i(t-\tau) + f(t, z_t) - KH\Delta(t)[\Theta(z(t)) + E_1 z_1(t-\tau(t))] - \\ &\quad [F_M(t, z_t) + D_M(z_t) + \|KH\|\, \|\Theta(z(t))\| + M]\frac{s}{\|s\|})\end{aligned}$$

then we derive that:

$$\dot{V} \leq -2M\, \|s(z(t)\| = -2MV(t)^{\frac{1}{2}}.$$

This last inequality is known to prove the finite-time convergence of the system (20) into the surface $s = 0$ ([18]).

Stability of the reduced system:

On the sliding manifold $s(z) = 0$, the system is driven by the following reduced system:

$$\frac{dz_1(t)}{dt} = (A_{11} + A_{12}K + H\Delta(t)(E_0 + E_2K))z_1(t) + (A_{d11} + A_{d12}K + H\Delta(t)E_1)z_1(t-\tau(t)) \quad (27)$$

According to Theorem 1, this system is asymptotically stable for any delay law $\tau(t) \leq h$ if, for some positive number ε, there exist positive numbers $\bar{\delta}_1, \bar{\delta}_2$ and matrices $0 < Q_1,\ Q_2,\ Q_3,\ Y \in \mathcal{R}^{m\times(n-m)}$ that satisfy the LMI (24). ♣

Remark 1 *Note that the explicit knowledge of the time-dependance of the delay is not required in the expression of the control law $u(t)$, all is needed is the knowledge of an upper bound h.*

4 Example

We demonstrate the applicability of the above theory by solving the example from [9] for a system without uncertainty. Consider system

$$\dot{x}(t) = Ax(t) + A_d x(t-\tau) + B[u(t) + f(x,t)], \quad (28)$$

	delay upper bound	type of delay
Theorem 2	3.999	time-varying
Gouaisbaut et al [9]	1.65	constant
Ivanescu et al.[10]	1.46	constant
Fu et al.[8]	0.984	constant
Li and de Souza[14]	0.51	constant

Table 1: Comparison of results for example (27)-(28)

with a time-varying delay, where

$$A = \begin{bmatrix} 2 & 0 \\ 1.75 & 0.25 \end{bmatrix}, A_d = \begin{bmatrix} -1 & 0 \\ -0.1 & -0.25 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{29}$$

By an appropriate change of variables, this system is equivalent to:

$$\dot{z}(t) = \tilde{A}z(t) + \tilde{A}_d z(t-\tau) + \tilde{B}[u(t) + f(x,t)],$$

where

$$\tilde{A} = \begin{bmatrix} 0.25 & 0 \\ 1.75 & 2 \end{bmatrix}, \tilde{A}_d = \begin{bmatrix} -0.9 & -0.65 \\ -0.1 & -0.35 \end{bmatrix}, \tilde{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{30}$$

As the pair $(\tilde{A}_{11}, \tilde{A}_{12})$ is not controllable, the system cannot be stabilized independently of the delay.

For this system, previous published works give the following results:

— In the case of a constant delay and $f = 0$, the system may be stabilized using a linear memoryless controller $u(t) = Kx(t)$ for the following maximum values of h: $h = 0.51$ by [14], $h = 0.984$ by [8] and $h = 1.46$ by [10]. By sliding mode control for the case of constant delay and $f \neq 0$ the maximum value of $h = 1.65$.

— Applying Theorem 2 in the case of a time-varying delay and $f \neq 0$, the corresponding value of $h = 3.999$ is achieved.

This is summarized in table 1.

5 Conclusions

The problem of finding a sliding mode controller that asymptotically stabilizes a system with time-varying delay and norm-bounded uncertainty has been solved. A delay-dependent solution has been derived using a special Lyapunov-Krasovskii functional. The result is based on a sufficient condition and it thus entails an overdesign. This overdesign is considerably reduced due to the fact that the method is based on the descriptor representation. As a byproduct for the first time on the basis of the descriptor model transformation the solution to the stabilization problem by the feedback, which depends on both, non-delayed and delayed state is solved. Finally, a numerical example shows the effectiveness of the combined method: sliding mode and descriptor representation.

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