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ROBUST STABILITY OF LARGE-SCALE SYSTEMS WITH AFTEREFFECT

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ABSTRACT

This article is devoted to the robust stability analysis of large-scale systems involving both discrete and distributed delays. Here, the robustness property is relative to uncertainties on the time-delay function as well as on model parameters. Differential inequalities and comparison principle lead to consider a simpler system, which stability implies the stability of original one. An application to Hopfield's neural networks illustrates the work.

Key words: Asymptotic stability, Attractors, Delay, Dynamic systems, Interconnected systems, Large-scale systems, Neural networks, Robust stability, Stability criteria.

1 INTRODUCTION

The major part of the engineering systems involve several interconnected subsystems, possibly coming from different physical domains. Examples of such composite, large scale or interconnected systems are classically encountered in the control of electric power networks, nuclear reactors, aerospace industries, chemical and petroleum plants,.. but also in other sciences such as economy (interaction between sectors of economy (Leontief, *et al*, 1953)), life sciences, societal or ecological systems (see (Michel and Miller, 1977) and references therein for more details).

Besides, systems with aftereffect are also involved in engineering: such models permit one to take into account the phenomena of memory and material, energy or information transmission (Kolmanovskii and Myshkis, 1992; Kolmanovskii and Nosov, 1986; Hale, 1977): this kind of phenomena has a great influence on the behavior of interconnected systems, in particular when considering the stability property which is known to be one of the major questions encountered in control applications.

Such stability study generally involves some Lyapunov functions in the ordinary differential equations (ODE) case, and functional (Krasovskii's approach Krasovskii, 1963) or functions (Razumikhin's approach, Razumikhin, 1960) in the functional differential equations (FDE) case. But the way to generate them is of course not obvious.

The other approach we use in this paper is based on differential inequalities and comparison principles: it consists in studying a simpler *comparison* system, which stability implies the stability of the original system. It has been widely considered in stability analysis of ODE (see for instance Lakshmikantham and Leela, 1969; Perruquetti *et al*, 1995), and then generalized to retarded systems (see Barholoméüs, 1996; Richard *et al*, 1997) and neutral systems (Tchangani *et al*, 1997).

Here, this concept will allow us to derive sufficient robust stability conditions for some time-delayed large-scale systems. In the literature, robustness is often considered with regard to model uncertainties (Niculescu *et al*, 1994; Michel and Wang, 1995) or with regard to time-delay (Richard *et al*, 1997; Verriest, 1994). In what concerns this last, some compromise has to be found: it is worth deriving independent-of-delay (i.o.d.) stability, which is a very delay-robust property but needs very conservative conditions; besides, delay-dependent (d.d.) stability yields more "realistic" conditions but needs some known bounds of the delay variations. In particular, d.d. stability is the only property that can be checked for non-delayed unstable open-loop process with a purely delayed feedback control. Then, the present paper considers robustness with regard to both parameters and delays: it provides both d.d. and i.o.d. results.

Let $x_i(t) \in R^{n_i}$, with norm $||.||_i$ be the current value of the state function $x_{i_t} \in C_i = C([-h,0]; R^{n_i})$ of subsystem i defined by: $x_{i_t}(\theta) = x_i(t+\theta) \forall \theta \in [-h,0]$, and $x = [x_1^T, .., x_i^T, .., x_r^T]^T, C = \prod_{i=1}^r C_i$; we consider here uncertain time-delay systems of the form,

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + \Delta a_i(t, x_t) + \\ & B_i x_i(t - h(t, x_t)) + \Delta b_i(t, x_t) + \\ & \int_{-\tau(t,x_t)}^{0} [C_i x_i(t+s) + \Delta c_i(s, x_t(s))] ds, \end{aligned} \tag{1}$$

$t \geq t_0,\ i = 1, ..., r\ ,\ x_{i_{t_0}}(\theta) = \varphi(\theta)\ \forall \theta \in [-h, 0]$, where

- $h(t, x_t)$ and $\tau(t, x_t)$ are piecewise continuous functional verifying: $0 \leq h(t, x_t) \leq h_m$, $0 \leq \tau(t, x_t) \leq \tau_m$, $h = \max\{h_m, \tau_m\}$;
- A_i, B_i, C_i are constant, $n_i \times n_i$ matrices;
- the uncertainties $(\Delta a_i(t, x_t),\ \Delta b_i(t, x_t),\ \Delta c_i(s, x_t(s)))$ verify

$\Delta a_i(t, x_t) = \sum_{j=1}^r A_{ij}(t, x_t) x_j(t)$,
$\Delta b_i(t, x_t) = \sum_{j=1}^r B_{ij}(t, x_t) x_j(t - h(t, x_t))$,
$\Delta c_i(s, x_t(s)) = \sum_{j=1}^r C_{ij}(s, x_t(s)) x_j(t+s)$;
with conditions, $\forall\ x_t \in C$:
$||A_{ij}(.) x_j(t)||_i \leq \alpha_{ij}(.) ||x_j(t)||_j$
$||B_{ij}(.) x_j(t - h(.))||_i \leq \beta_{ij}(.) ||x_j(t - h(.))||_j$
$||C_{ij}(.) x_j(t+s)||_i \leq \gamma_{ij}(.) ||x_j(t+s)||_j$
Let us define following sets:

- $\Omega_i(q_i) = \{x_i \in R^{n_i} : ||x_i||_i \leq q_i\}, q_i > 0$, a 0-centered ball of R^{n_i} with radius q_i;
- $C(\Omega_i(q_i)) = \{\varphi_i \in C_i : \varphi_i(\theta) \in \Omega_i(q_i), \theta \in [-h, 0]\}$;
- $\Omega(q) = \prod_{i=1}^r \Omega_i(q_i)$ and $C(\Omega(q)) = \prod_{i=1}^r C(\Omega_i(q_i)), q = [q_1, .., q_i, .., q_r]^T$;

$\mu(X)$ is the matrix measure of the matrix X (Bartholoméüs, 1996) associated with the norm $||.||$,

$$\mu(X) = \lim_{\varepsilon \to 0^+} \frac{||I + \varepsilon X|| - 1}{\varepsilon}.$$

2 Main results

This section is divided in two parts: the first one defines the main tool used for stability analysis and gives independent of discrete delay stability criteria for system (1); in the second subsection, a model transformation is used so to take into account the influence of the discrete delay value on the stability.

2.1 Independent-of-discrete-delay conditions

Let us set:

- $F(t, x_t) = [F_{ij}]$ with $F_{ii} = \mu_i(A_i) + \alpha_{ii}(t, x_t)$ and $F_{ij} = \alpha_{ij}(t, x_t),\ j \neq i$;
- $G(t, x_t) = [G_{ij}]$ with $G_{ii} = ||B_{0i}||_i + \beta_{ii}(t, x_t)$ and $G_{ij} = \beta_{ij}(t, x_t),\ j \neq i$;
- $H(s, x_t(s)) = [H_{ij}]$ with $H_{ii} = ||C_i||_i + \gamma_{ii}(s, x_t(s))$ and $\gamma_{ij}(s, x_t(s)),\ j \neq i$;
- $||x(t)|| = [||x_1(t)||_1, .., ||x_i(t)||_i, .., ||x_r(t)||_r]^T$;

then the following lemma holds.

Lemma 2.1 *The following inequality is satisfied along any solution of (1):*

$$\begin{aligned} D^+||x(t)|| \leq\ & F(t, x_t)||x(t)|| + \\ & G(t, x_t)||x(t - h(t, x_t))|| \\ & + \int_{-\tau(t,x_t)}^{0} H(s, x_t(s))||x(t+s)||ds. \end{aligned} \tag{2}$$

(D^+ is the Dini derivative).

Proof. The proof is based on results in (Tchangani *et al*, 1997a). Let us consider for any $i \in \{1, 2, ..., r\}$ as in (Kolmanovskii, 1995), the function $Q_i : R^{n_i} \times R^{n_i} \longrightarrow R$ defined by

$$Q_i[x_i, y_i] = \lim_{\Delta \to 0^+} \frac{1}{\Delta} [||x_i + \Delta y_i||_i - ||x_i||_i] \tag{3}$$

then we have $D^+ ||x_i(t)||_i = Q_i[x_i(t), \dot{x}_i(t)]$. Replacing $\dot{x}_i(t)$ by its value in (1) and developing, yields the inequality

$$\begin{aligned} D^+ ||x_i(t)||_i \leq\ & Q_i[x_i(t), A_i x_i(t)] + \\ & \sum_{j=1}^n \alpha_{ij}(t, x_t)||x_j(t)||_j + \\ & ||B_i||_i ||x_i(t - h(t, x_t))||_i + \\ & \sum_{j=1}^n \beta_{ij}(t, x_t))||x_j(t - h(t, x_t))||_j + \\ & \int_{-\tau(t,x_t)}^{0} [||C_i||_i ||x_i(t+s)||_i + \\ & \sum_{j=1}^n \gamma_{ij}(s, x_t(s))||x_j(t+s)||_j] ds. \end{aligned}$$

Doing this for all $i = 1, 2, ..., r$ and remarking that $\mu_i(A_i) = \sup_{x_i \neq 0} ||x_i||_i^{-1} Q_i[x_i, A_i x_i]$ completes the proof.

Now, let us associate to the inequality (2) the following system,

$$\dot{z}(t) = F(t,x_t)z(t) + G(t,x_t)z(t-h(t,x_t)) + \int_{-\tau(t,x_t)}^{0} H(s,x_t(s))z(t+s)ds, \qquad (4)$$

$z(t) \in R^r$; then one can derive the following important lemma, which states that the system (4) is a comparison system of (1).

Lemma 2.2 *If the solution of (4) exists and is unique for any solution $x(t)$ of (1) then the following property holds for $i = 1, 2, ...r$,*

$$\begin{aligned} \text{if} \quad ||\varphi_i(\theta)||_i &\leq z_i(t_0+\theta) \ \forall\theta \in [-h,0], \quad (5)\\ \text{then} \quad ||x_i(t)||_i &\leq z_i(t) \qquad \forall t \geq t_0. \end{aligned}$$

Proof. : obtained by applying Lemma 2.1 and results in (Lakshmikantham and Leela, 1969; Tchangani *et al*, 1997a).

On the basis of previous lemmas, we can now prove the asymptotic stability condition of (1) expressed in the following theorem.

Theorem 2.3 *The zero solution of (1) is asymptotically stable if there exists a positive vector u and a positive scalar ε such that:*

$$[F(t,x_t)+G(t,x_t)+\int_{-\tau(t,x_t)}^{0} H(s,x_t(s))ds]u < -\varepsilon u. \qquad (6)$$

Moreover, the set $C(\Omega(\lambda u))$ for any positive scalar λ is positively invariant.

Proof. : According to Lemma 2.2, it is sufficient to prove that the zero solution of (4) is asymptotically stable. For this, define the function

$$v(z(t)) = \max_i\{\frac{|z_i(t)|}{u_i}\}$$

and then establish as in (Tchangani, *et al*, 1997a) that this is a Lyapunov-Razumikhin function.

When the bounds of uncertainties are constant (which is the most classical case), the conditions are simplified as follows.

Corollary 2.4 *If the scalars α_{ij}, β_{ij}, γ_{ij} are constant and the matrix $F+G+\tau_m H$ is Hurwitz, then the zero solution of (1) is asymptotically stable.*

2.2 DEPENDENT-ON-DISCRETE-DELAY CONDITIONS

In the above criteria, the parameters that mainly ensure stability conditions are submatrices A_i. But, the non delayed ($h = \tau = 0$) and non perturbed system ($\Delta a_i(.) = \Delta b_i(.) = \Delta c_i(.) = 0$) corresponding to (1) is

$$\dot{x_i}(t) = (A_i + B_i)x_i(t), \qquad i = 1, ..., r,$$

which zero solution is asymptotically stable if the matrices $A_i + B_i$ is Hurwitz. According to this it seems important to take into account a possible stabilizing influence of matrices B_i, which in turn means to look for stability criteria depending on the discrete delay. In the following subsection, we introduce this dependency by using some transformation, classically based on the equality:

$$x_i(t-h(t,x_t)) = x_i(t) - \int_{-h(t,x_t)}^{0} \dot{x_i}(t+u)du, \qquad (7)$$

and so (1) can be developed, for $t \geq t_0 + h$, as:

$$\begin{aligned} \dot{x_i}(t) = \; & (A_i + B_i)x_i(t) + \Delta a_i(t,x_t) + \Delta b_i(t,x_t) \\ & + \int_{-\tau(t,x_t)}^{0} [C_i x_i(t+s) + \Delta c_i(s,x_t(s))]ds \\ & - B_i A_i \int_{-h(t,x_t)}^{0} x_i(t+u)du \\ & - B_i \int_{-h(t,x_t)}^{0} \Delta a_i(t+u,x_{t+u})du \\ & - B_i^2 \int_{-h(t,x_t)}^{0} x_i(t+u-h(t+u,x_{t+u}))du \\ & - B_i \int_{-h(t,x_t)}^{0} \Delta b_i(t+u,x_{t+u})du \\ & - B_i \int_{-h(t,x_t)}^{0} du \int_{-\tau(t+u,x_{t+u})}^{0} \\ & [C_i x_i(t+u+s) + \Delta c_i(s,x_{t+u}(s))]ds \qquad (8) \end{aligned}$$

Let us define matrices $K(t,x_t)$, $L(t,x_t)$, $M(t+u,x_{t+u})$, $N(t+u,x_{t+u})$ and $P(s,x_{t+u}(s))$ as:

- $K_{ii}(t,x_t) = \mu_i(A_i + B_i) + \alpha_{ii}(t,x_t)$ and $K_{ij}(t,x_t) = \alpha_{ij}(t,x_t)$, $j \neq i$;
- $L_{ij}(t,x_t) = \beta_{ij}(t,x_t)$;
- $M_{ii}(t+u,x_{t+u}) = ||B_iA_i||_i + ||B_i||_i\alpha_{ii}(t+u,x_{t+u})$ and $M_{ij}(t+u,x_{t+u}) = \alpha_{ij}(t+u,x_{t+u})$, $j \neq i$;
- $N_{ii}(t+u,x_{t+u}) = ||B_i^2||_i + ||B_i||_i\beta_{ii}(t+u,x_{t+u})$ and $N_{ij}(t+u,x_{t+u}) = \beta_{ij}(t+u,x_{t+u})$, $j \neq i$;
- $P_{ii}(s,x_{t+u}(s)) = ||B_iC_i||_i + ||B_i||_i\gamma_{ii}(s,x_{t+u}(s))$ and $P_{ij}(s,x_{t+u}(s)) = \gamma_{ij}(s,x_{t+u}(s))$, $j \neq i$;

then a system like (4) can be obtained for $t \geq t_0 + h$ as

$$\begin{aligned}\dot{z}(t) &= K(t,x_t)z(t) + L(t,x_t)z(t-h(t,x_t)) \\ &+ \int_{-h(t,x_t)}^{0} H(s,x_t(s))ds + \\ &\int_{-h(t,x_t)}^{0} M(t+u,x_{t+u})z(t+u)du + \\ &\int_{-h(t,x_t)}^{0} N(t+u,x_{t+u})z(t+u- \\ &h(t+u,x_{t+u}))du + \\ &\int_{-h(t,x_t)}^{0} du \int_{-\tau(t+u,x_{t+u})}^{0} \\ &P(s,x_{t+u}(s))z(t+u+s)ds \qquad (9)\end{aligned}$$

and we have the Lemma 2.5:

Lemma 2.5 *If the solution of (9) exists and is unique for any solution $x(t)$ of (1), then the system (9) is a comparison system of (1) (this means, verifies 5).*

Remark 1 : *In this case, the delay of the comparison system is twice the original delay.*

Proceeding as previously leads to Theorem 2.6 and Corollary 2.7.

Theorem 2.6 *The zero solution of (1) is asymptotically stable if there exists a positive vector u and a positive scalar ε such that:*

$$\begin{aligned}-\varepsilon u &> [K(t,x_t) + L(t,x_t) + \\ &\int_{-\tau(t,x_t)}^{0} H(s,x_t(s))ds + \\ &\int_{-h(t,x_t)}^{0} \{M(t+u,x_{t+u}) \\ &+N(t+u,x_{t+u})\}du + \int_{-h(t,x_t)}^{0} du \\ &\int_{-\tau(t+u,x_{t+u})}^{0} P(s,x_{t+u}(s))ds]u. \quad (10)\end{aligned}$$

Moreover, the set $C(\Omega(\lambda u))$ for any $\lambda > 0$ is positively invariant.

Corollary 2.7 *If the scalar bounds α_{ij}, β_{ij}, γ_{ij} are constant and the matrix $K+L+\tau_m H+h_m(M+N)+h_m\tau_m P$ is Hurwitz, then the zero solution of (1) is asymptotically stable.*

3 ASYMPTOTIC ATTRACTORS

We consider now the case where uncertainties are bounded by constants (and not by gains): $||\Delta a_i(t,x_t)||_i \leq \alpha_i$, $||\Delta b_i(t,x_t)||_i \leq \beta_i$, $||\Delta c_i(t,x_t)||_i \leq \gamma_i$, $\forall x_t \in C$.

In this case, it is not possible to derive a comparison system as (4)(9) by the previous method because the system one can derive will be non homogeneous: the state space origin is possibly not anymore an equilibrium. But, in this case it is possible to estimate an attractor of (1). This is important because, in practice it can only be sufficient for a system to behave in the vicinity of an operating point, without necessarily reaching it.

Definition 3.1 *A subset $C(\Omega(q))$ of C is a (global) attractor of (1) if all trajectories of (1) converge asymptotically towards $C(\Omega(q))$; that is*

$$\forall\, \varepsilon > 0,\ \exists\, T \geq 0,\ \forall\, t \geq t_0 + T,\ d(x_i(t), \Omega_i(q_i)) \leq \varepsilon$$

where $x(t)$ is a solution of (1) with $d(x_i(t),\Omega_i(q_i)) = \inf_{y_i\in\Omega_i(q_i)} \|x_i(t) - y_i\|$.

We shall only consider the independent-of-discrete-delay case; to obtain the results depending on discrete delay the procedure is similar to Section 2.2. Without lost of generality we consider that the delays are constant, equal respectively to h_m and τ_m. In this case the comparison system is completely disconnected into:

$$\begin{aligned}\dot{z}_i(t) &= \gamma_i(A_i)z_i(t) + ||B_i||_i z_i(t-h_m) + \\ &||C_i||_i \int_{-\tau_m}^{0} z_i(t+s)ds + \alpha_i + \beta_i + \tau_m\gamma_i,\end{aligned}$$

$i = 1, 2, .., r$ and so, Theorem 3.1 follows.

Theorem 3.1 *If*

$$\gamma_i(A_i) + ||B_i||_i + \tau_m||C_i||_i < 0$$

then the set $C(\Omega(q))$ where $q = [q_1, .., q_i, .., q_r]^T$ with

$$q_i = -\frac{\alpha_i + \beta_i + \tau_m\gamma_i}{\gamma_i(A_i) + ||B_i||_i + \tau_m||C_i||_i}$$

is an asymptotically stable attractor of (1).

4 APPLICATION TO ARTIFICIAL NEURAL NETWORKS

Figure 1 represents a Hopfield neural network (Michel and Wang, 1995), where $R_i, C_i > 0$ denote respectively resistance and capacitance of the i^{th}

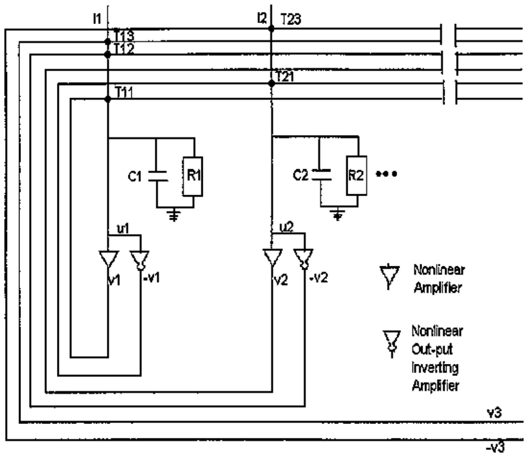

Figure 1 – Hopfield neural network model

unit, $T_{ij} = \frac{1}{R_{ij}}$, R_{ij} represent the interconnection resistance between the output of unit i and the input of j (R_{ij} can be negative because of sign inversions).

During the implementation process of artificial neural network by VLSI for instance , the introduction of time delay is unavoidable. So, the dynamic equation of the network is (see Michel and Wang, 1995)

$$\dot{x}(t) = -Ax(t) + TS(x(t-h)) + I(t) \qquad (11)$$

where h is the transmission delay with $x(t) = [.., x_i(t), ..]^T$, $x_i(t) = C_i u_i(t)$; $A = diag\{a_i\}$, $i = 1, 2, ..n$, $a_i = \frac{1}{\tau_i C_i} > 0$; $\frac{1}{\tau_i} = \frac{1}{R_i} + \sum_{j=1}^{n} |T_{ij}|$; $T = [T_{ij}]$; $S(x) = [.., s_i(x_i), ..]^T$, $s_i(x_i) = g_i(\frac{u_i}{C_i})$; $v_i(t) = g_i(u_i(t))$. Function $g_i \in C(R, (-1, 1))$ corresponds to a nonlinear amplifier: it is strictly increasing with $g_i(0) = 0$, $u_i g_i(u_i) > 0$. We consider it is differentiable, however this is not necessary (see remark). Lastly, $I = [.., I_i, ..]^T$, $I_i(t) \in C(R^+, R)$ is the external input.

Artificial neural networks as (11) are used in many areas as image processing, pattern recognition, optimization, etc. They are also used as associative memories: in this case, the external sources I_i are usually assumed to be constant functions, and asymptotically stable equilibria of (11) are used as memories. Besides, it is shown on the basis of linearized equation of (11) that the system can be either i.o.d. stable, or there exists a delay such that the system becomes unstable (see for instance (Niculescu, 1996) and references therein). In the following, we will apply previous results by considering nonlinear part of equation (11) as a perturbation, so to derive stability conditions of the equilibria of (11).

We will consider without loss of generality the autonomous case of (11), that is,

$$\dot{x}(t) = -Ax(t) + TS(x(t-h)) \qquad (12)$$

and give stability condition of the zero equilibrium.

Let us set $D(x) = \frac{\partial S(x)}{\partial x}$, $\beta_i = \sup_{x \in R^n} |D_i(x)|$, $\beta = \sup_i\{\beta_i\}$ and $N = diag\{\beta_i\}$; for a given matrix $M \in R^{n \times n}$, $|M|$ denotes the matrix with absolute value of entries of M. We can then prove the following proposition.

Proposition 1 *If there exist a vector $u > 0$ and a scalar $\varepsilon > 0$ such that*

$$[-A + \int_0^1 |TD(\eta x(t-h))| \, d\eta]u < -\varepsilon u$$

then the zero solution of (12) is asymptotically stable

Proof. Note that according to the fact that $S(0) = 0$, (12) can be rewritten as

$$\dot{x}(t) = -Ax(t) + \left[T \int_0^1 D(\eta x(t-h)) d\eta\right] x(t-h).$$

and then apply Theorem 2.3.

The following corollary gives an easy-to-check criterion.

Corollary 2 *If there exists a matrix $N^* = diag\{\beta_i^*\}$ with $\beta_i^* \geq \beta_i$, $i = 1, 2, ..n$ such that $-A + |T| N^*$ is Hurwitz, then zero solution of (12) is asymptotically stable.*

In particular, if

$$\beta < \frac{1}{C_i}\left(1 + \frac{1}{R_i \sum_{j=1}^{n} |T_{ij}|}\right), \; i = 1, 2, ..., n, \qquad (13)$$

then the zero solution of (12) is asymptotically stable.

Remark 2 *Note that if $S(x)$ is not differentiable, previous corollary can be applied if the condition $0 \leq \left|\frac{s_i(x_i)}{x_i}\right| \leq \beta_i^*$ holds; this is the condition required in (Michel and Wang, 1995). Condition (13) can be used to design parameters of a neural networks circuit.*

5 CONCLUSION

The robust stability of large scale systems with aftereffect has been considered in relation to both uncertainties of model and time delay functions. I.o.d. results were proposed, as well as d.d. ones. The depend on the size of the discrete delays was only considered, but a similar study can be done on the basis of the distributed delays size. In the (classical) case where uncertainties are bounded by constants, an asymptotically stable attractor was estimated, depending on the size of the delay. This can be important in practice because it gives an idea of the reachable target with regard to delay. A practical example is studied to show the efficiency of the results given in the paper.

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