ROBUST STABILITY AND STABILIZATION OF SYSTEMS WITH AFTEREFFECT

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Abstract. This paper considers the robust, stabilizing control of time-delay, linear systems with nonlinear uncertainties on the parameters and possible variations of the delays. It first gives some stability criteria for systems with both distributed and discrete delays, and then applies them to the design of input-delayed controllers. The proofs are based on comparison methods.

Keywords: stability, stabilization, robustness, systems with aftereffect

1 Introduction and statement of the problem

The aftereffect phenomenon is a natural component of the dynamic processes in many engineering fields (see for instance [4, 5]). Even if the process itself does not include delay phenomena in its inner dynamics, the actuators, sensors and transmission lines that are involved in its automatic control usually introduce such time lags. This explains the great number of works devoted to the so-called functional differential equations (shortly, *FDEs*) [15].

Two generalizations of the Lyapunov direct method, by Razumikhin and Krasovskii, are available for FDEs and still receive a great interest (see [3]); they either involve the construction of a definite positive *function* $v(x(t))$ (Razumikhin theory) or of a *functional* $v(x_t)$. For robust stability criteria, the Krasovskii approach is mainly chosen, using some generalized quadratic-type functionals: this leads to Riccati-type equations, which in turn are to be studied by computational algorithms such as LMI (linear matrix inequalities) approach. In the multi-delay case, the obtained LMIs have high dimension (see for instance [3]). The distributed-delay case was rarely considered through this approach [6].

The *comparison systems* constitute another option: the idea is to derive stability of original system by studying stability of a simpler system, obtained by using differential inequalities and vector Lyapunov functions (see [7, 8] for the general theory and [1, 16] for concrete choices of vector-Lyapunov functions with Razumikhin principle). It has shown some advantages in basic stability analysis, but also for the estimation of the stability domains (see [16] and references herein). The results apply to the stability study of retarded systems with discrete [1] or distributed delays [18] and to neutral ones [19].

This paper applies this concept to the robust stability and stabilization of uncertain, mixed discrete plus-distributed delay systems,

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bx(t-\eta(t)) + f(t,x(t)) + g(t,x(t-\eta(t))) \\ &\quad + \int_{-\tau(t)}^{0} [Cx(t+s) + h(s,x(t+s))]ds, \\ t &\geq t_0,\ x_{t_0}(\theta) = \varphi(\theta),\ \theta \in [-\xi, 0]. \end{aligned} \tag{1}$$

Here, the delays $\eta(t)$ (discrete) and $\tau(t)$ (distributed) are piecewise continuous functions verifying: $0 \leq \eta(t) \leq \eta_m$ and $0 \leq \tau(t) \leq \tau_m$. $A, B, C \in \mathbb{R}^{n\times n}$ are constant matrices, and the nonlinear functions f, g and h are uncertainties which can be structured or not ([1], see definitions in the additional notations).

In the existing literature (see a recent survey in [3]), robustness is often considered with regard to model uncertainties [13, 12, 11, 9, 1] or with regard to time-delay [21, 1]. In what concerns this last, independent-of-delay *(i.o.d.)* stability criteria and delay-dependent *(d.d.)* ones are generally distinguished: i.o.d. stability is a very strong property, since it holds for any value of the delay, but needs restrictive conditions. Delay-dependent conditions are generally preferable since, in practice, delay upper-bounds are known.

But, to the best authors knowledge, there are no simple result that concern robust stability of distributed-delay systems, even if this kind of operators is basic in finite-spectrum assignment of linear, time-invariant systems with constant delays [2][22]. Hence, we hope the present study constitutes a possible approach to the robustness analysis of such controllers: robustness with regard to variations of the delay, and with regard to some additive, nonlinear uncertainties.

The paper is organized as follows: Section 2 defines robust, delay-dependent stability criteria. A theorem and its corollary are proved for structured uncertainties, and statements in the unstructured case follow without proof. Then, Section 3 uses these results for the stabilizing control of systems with delayed state and input.

1.1 Additional notations

$x(t) \in \mathbb{R}^n$, with norm $||.||$, is the instantaneous value of the state function $x_t \in \mathcal{C} = C([-\xi, 0]; \mathbb{R}^n)$, $\xi = \max\{\eta_m, \tau_m\}$, defined by: $x_t(\theta) = x(t+\theta)\ \forall\ \theta \in [-\xi, 0]$;

$|y| \in \mathbb{R}^n_+$ and $|M| \in \mathbb{R}^{n\times n}_+$ are the component-to-component absolute values of $y \in \mathbb{R}^n$ and $M \in \mathbb{R}^{n\times n}$.

M^* denotes the matrix with the same diagonal entries as $M \in \mathbb{R}^{n\times n}$ but with off-diagonal absolute values ($m^*_{ii} = m_{ii}$, $m^*_{ij} = |m_{ij}|$).

$\mu(M) = lim_{\varepsilon\to 0^+} \dfrac{||I + \varepsilon M|| - 1}{\varepsilon}$, the matrix measure of M, and $\|M\| = \sup\limits_{y\neq 0} \dfrac{\|My\|}{\|y\|}$.

Structured uncertainties [1]: one can estimate the component-by-component effect of perturbations. Then, $\forall\ t \in \mathbb{R}$, $\forall\ x \in \mathbb{R}^n$:

$$\begin{aligned} |f(t,x)| &\leq F(t)\,|x|; \quad F(t) \in \mathbb{R}^{n\times n}_+, \\ |g(t,x)| &\leq G(t)\,|x|; \quad G(t) \in \mathbb{R}^{n\times n}_+, \\ |h(s,x)| &\leq H(s)\,|x|; \quad H(s) \in \mathbb{R}^{n\times n}_+. \end{aligned}$$

Unstructured uncertainties: only a global estimation of the influence of uncertainties can be obtained, and then, $\forall\ t \in \mathbb{R}$, $\forall\ x \in \mathbb{R}^n$:

$$\begin{aligned} \|f(t,x)\| &\leq \alpha(t)\,\|x\|; \quad \alpha(t) \in \mathbb{R}_+, \\ \|g(t,x)\| &\leq \beta(t)\,\|x\|; \quad \beta(t) \in \mathbb{R}_+, \\ \|h(s,x)\| &\leq \gamma(s)\,\|x\|; \quad \gamma(s) \in \mathbb{R}_+. \end{aligned}$$

2 Stability

Many results concern the case where the matrix A is stable [9, 12, 14]; this restriction was partially solved by considering the stabilizing influence of matrix B [3, 17]. This section completes it by including the stabilizing influence of a distributed-delay effect (matrix C).

2.1 Structured uncertainties

In the case of structured uncertainties, a matricial stability criterion for system (1) can be derived as expressed in the following theorem.

Theorem 1 *The zero solution of (1) is asymptotically stable if there are a vector $p > 0$ and a scalar $\varepsilon > 0$ such that for $t \geq t_0 + \xi$, $\mathbf{M}(t)p < -\varepsilon p$ holds for the matrix $\mathbf{M}(t)$ defined by*

$$\begin{aligned}
&(A+B+\tau(t)C)^* + F(t) + G(t) + \int_{-\tau(t)}^{0} H(s)ds \\
&+\eta(t)(|BA| + |B^2|) + |B| \int_{-\eta(t)}^{0} (F(t+u) + G(t+u))du \\
&+|BC| \int_{-\eta(t)}^{0} \tau(t+u)du + |B| \int_{-\eta(t)}^{0} \int_{-\tau(t+u)}^{0} H(s)ds\, du \\
&+\frac{\tau^2(t)}{2}(|CA| + |CB|) + |C| \int_{-\tau(t)}^{0} \int_{s}^{0} [F(t+u) + G(t+u)]du\, ds \\
&+|C^2| \int_{-\tau(t)}^{0} ds \int_{s}^{0} \tau(t+u)du + |C| \int_{-\tau(t)}^{0} \int_{s}^{0} \int_{-\tau(t+u)}^{0} H(v)\, dv\, du\, ds
\end{aligned} \tag{2}$$

When all uncertainties have constant upper-bound gains and delays are constant, the conditions of previous theorem become easier to check:

Corollary 2 *If F, G, H, η and τ are constant, then the zero solution of (1) is asymptotically stable if the following, constant matrix $\mathbf{M}$ is of Hurwitz-type*[1]*:*

$$\begin{aligned}
&(A+B+\tau C)^* + F + G + \tau H + \eta\{|BA| + |B^2| + |B|\,(F+G)\} + \\
&+\eta\tau(|BC| + |B|\,H) + \frac{\tau^2}{2}\{|CA| + |CB| + |C|\,(F+G)\} + \frac{\tau^3}{2}\{|C^2| + |C|\,H\}
\end{aligned} \tag{3}$$

Proof. Firstly, note that

$$x(t-\eta(t)) = x(t) - \int_{-\eta(t)}^{0} \dot{x}(t+u)du, \tag{4}$$

$$x(t+s) = x(t) - \int_{s}^{0} \dot{x}(t+u)du. \tag{5}$$

Thus (1) can be developed for $t \geq t_0 + \xi$ into

$$\begin{aligned}
&\dot{x}(t) = [A+B+\tau(t)C]x(t) + f(t,x(t)) + g(t,x(t\text{-}\eta(t))) + \textstyle\int_{-\tau(t)}^{0} h(s,x(t+s)]ds \\
&-\textstyle\int_{-\eta(t)}^{0} (BAx(t+u) + B^2x(t+u\text{-}\eta(t+u)) + Bf(t+u,x(t+u)) + Bg(x(t+u\text{-}\eta(t+u))) \\
&+\textstyle\int_{-\tau(t+u)}^{0} [BCx(t+s)+Bh(s,x(t+s))]ds)du - \int_{-\tau(t)}^{0} \{\int_{s}^{0} (CAx(t+u)+CBx(t+u\text{-}\eta(t+u)) \\
&+Cf(t+u,x(t+u))+Cg(x(t+u\text{-}\eta(t+u))) + \textstyle\int_{-\tau(t+u)}^{0} [C^2x(t+v)+Ch(v,x(t+v))]dv)du\}ds.
\end{aligned} \tag{6}$$

By using the vector-Lyapunov function $V(x(t)) = [|x_1(t)|, .., |x_i(t)|, .., |x_n(t)|]^T$, one can derive a comparison system of (6) as in [1, 17, 18], leading to the result.

[1] Remark that this equivalently means that (3) is the opposite of an M-matrix, since it has positive off-diagonal entries: hence, its stability can be simply checked by calculating the signs of its principal minors.

2.2 Unstructured uncertainties

Unstructured uncertainties leads to scalar stability criterion.

Theorem 3 *The zero solution of (1) is asymptotically stable if there is an $\varepsilon > 0$ such that*

$$\begin{aligned}
\sup_{t \geq t_0+\xi} &\{\mu(A+B+\tau(t)C) + \alpha(t) + \beta(t) + \int_{-\tau(t)}^{0} \gamma(s)\, ds + \eta(t)(\|BA\| + \|B^2\|) \\
&+\|B\| \int_{-\eta(t)}^{0} (\alpha(t+u) + \beta(t+u))\, du + \|BC\| \int_{-\eta(t)}^{0} \tau(t+u)\, du \\
&+\|B\| \int_{-\eta(t)}^{0} \int_{-\tau(t+u)}^{0} \gamma(s)\, du\, ds + \frac{\tau^2(t)}{2}(\|CA\| + \|CB\|) \\
&+\|C\| \int_{-\tau(t)}^{0} \int_{s}^{0} [\alpha(t+u) + \beta(t+u)]\, du\, ds + \|C^2\| \int_{-\tau(t)}^{0} \int_{s}^{0} \tau(t+u) du\, ds \\
&+\|C\| \int_{-\tau(t)}^{0} \int_{s}^{0} \int_{-\tau(t+u)}^{0} \gamma(v) dv\, du\, ds\} < -\varepsilon
\end{aligned} \tag{7}$$

Corollary 4 *If α, β, γ, η and τ are constant, then the zero solution of (1) is asymptotically stable if there exists $\varepsilon > 0$ such that*

$$\begin{aligned}
-\varepsilon \;>\; &\mu(A+B+\tau C) + \alpha + \beta + \tau\gamma + \eta\{\|BA\| + \|B^2\| + \|B\|\,(\alpha+\beta)\} + \\
&\eta\tau(\|BC\| + \|B\|\,\gamma) + \frac{\tau^2}{2}\{\|CA\| + \|CB\| + \|C\|\,(\alpha+\beta)\} + \frac{\tau^3}{2}\{\|C^2\| + \|C\|\,\gamma\}.
\end{aligned} \tag{8}$$

2.3 Generalization to multiple delays

The previous results are generalizable to the multiple-delay case. For instance, consider the following system,

$$\dot{x}(t) = \sum_{i=1}^{r} \int_{-\tau_i}^{0} [C_i x(t+s) + h_i(s, x(t+s))]\, ds. \tag{9}$$

with constant uncertainties bound, then it follows that

Theorem 5 *The zero solution of (9) is asymptotically stable if one of the following conditions holds:*

1) *the uncertainties are structured and*

$$\left(\sum_{i=1}^{r} \tau_i C_i\right)^* + \sum_{i,j=1}^{r} (|C_i C_j| + |C_i|\,H_j)\, \frac{\tau_i^2 \tau_j}{2} + \sum_{i=1}^{r} H_i \tau_i \text{ is Hurwitz;} \tag{10}$$

2) *the uncertainties are unstructured and*

$$\mu\left(\sum_{i=1}^{r} \tau_i C_i\right) + \sum_{i=1}^{r} \gamma_i \tau_i + \sum_{i,j=1}^{r} \left(\|C_i C_j\| + \|C_i\|\,\gamma_j\right) \frac{\tau_i^2 \tau_j}{2} < 0. \tag{11}$$

Let us consider now the following system ($0 < \tau_2 < \tau_1$)

$$\dot{x}(t) = \int_{-\tau_1}^{-\tau_2} [Cx(t+s) + h(s, x(t+s))]\, ds; \tag{12}$$

it will allow one to design controllers as (19). These equations represent the model of systems with distributed delay followed by a discrete delay. Indeed, let put

$$y(t) = C\int_{t-(\tau_1-\tau_2)}^{t} x(s)ds$$

then

$$\dot{x}(t) = y(t-\tau_2)$$

Using transformation (5) and putting $\rho = \frac{1}{2}(\tau_1^2 - \tau_2^2)$ it follows that:

Theorem 6 *The zero solution of (12) is asymptotically stable if one of the following conditions holds:*
1) *the uncertainties are structured and*

$$C^* + \rho\left(|C^2| + |C|\,H\right) + H \text{ is Hurwitz;} \tag{13}$$

2) *the uncertainties are unstructured and if*

$$\mu(C) + \rho\left(\|C^2\| + \|C\|\,\gamma\right) + \gamma < 0. \tag{14}$$

Proof. Analogous to the previous ones but considering a supremum over the time interval $[-\tau_1, -\tau_2]$.

3 Stabilization

Most of the existing literature concerning the robust stabilization of

$$\dot{x}(t) = Ax(t) + Bx(t-\eta(t)) + f(t,x(t)) + g(x(t-\eta(t)) + D(t)u(t) \tag{15}$$

(where $u(t) : \mathbb{R} \to \mathbb{R}^p$, piecewise continuous is the control , $D : \mathbb{R} \to \mathbb{R}^{n\times p}$ continuous), consider a *memoryless* control $u(t) = k(x(t))$ [9, 11, 12, 14]. It means that the parameter with crucial importance is the matrix A which must be stabilizable (in the case D constant for example): if the pair (A, D) is not stabilizable, there will be no answer for the stabilization problem, even if the pair $(A + B, D)$ (which corresponds to the case of negligible delay) is stabilizable. Moreover, memoryless control is not so much realistic in our opinion, since in many cases the delay is introduced by the measurement or control devices, leading to $u(t) = k(x(t-\tau))$ [10]. Then, the previous results appear as useful complements for the robust stabilization of (15) by means of a delayed control. We will give in the following two delayed feedback control laws as basic to stabilize the previous systems by basic applications of previous results.

3.1 Discrete-delayed control

Let us firstly consider the following controller, applied on system (15),

$$u(t) = K(t)x(t-\tau(t)). \tag{16}$$

where $K(t)$ is piece-wise continuous, and $\tau(t)$ verifies $0 \le \tau(t) \le \tau_m$. The following result is a direct application of [17, 18].

Proposition 7 *The system (15) is robustly stabilized by (16) if, for some scalar $\varepsilon > 0$, one of the following conditions holds for $t \ge t_0 + \xi$,*
1) *in the case of structured uncertainties, there is a vector $p > 0$, such that $\mathbf{M}(t)p < -\varepsilon p$ holds for the matrix $\mathbf{M}(t)$ defined by*

$$\begin{aligned}(A+B+D(t)K(t))^* + F(t) + G(t) + \eta(t)\{|BA| + |B^2| + |B|\,(F(t)+G(t))\\ + |BD(t)K(t)|\} + \tau(t)\{|D(t)K(t)A| + |D(t)K(t)|\,(F(t)+G(t)) + |D(t)K(t)B|\\ + |(D(t)K(t))^2|\}\end{aligned} \tag{17}$$

2) *in the case of unstructured uncertainties,*

$$\begin{aligned}\mu(A+B+D(t)K(t)) + \alpha(t) + \beta(t) + \eta(t)\{\|BA\| + \|B\|\,(\alpha(t)+\beta(t))\\ + \|B^2\| + \|BD(t)K(t)\|\} + \tau(t)\{\|D(t)K(t)A\| + \|D(t)K(t)\|\,(\alpha(t)+\beta(t))\\ + \|D(t)K(t)B\| + \|(D(t)K(t))^2\|\} < -\varepsilon.\end{aligned} \tag{18}$$

3.2 Distributed delayed control

Now consider the following control law, involving a distributed-delay effect

$$u(t) = \int_{-\tau(t)}^{-\nu(t)} Kx(t+s)ds \tag{19}$$

where $\tau_m \ge \tau(t) > \nu(t) \ge 0\ \forall t$, and $K =$ constant. Application of the results of Section 2 to system (15) with the control law (19), yields the following proposition.

Proposition 8 *The system (15) is robustly stabilized by (19) if, for some scalar $\varepsilon > 0$, one of the following conditions holds for $t > t_0 + \xi$,*
1) *in the case of structured uncertainties, there is a vector $p > 0$, such that $\mathbf{M}(t)p < -\varepsilon p$ holds for the matrix $\mathbf{M}(t)$ defined by*

$$\begin{aligned}\{A+B+[\tau(t)-\nu(t)]D(t)K\}^* + F(t) + G(t) + \eta(t)\left\{|BA| + |B^2|\right\} + |B|\int_{-\eta(t)}^{0}(F(t+u)\\ +G(t+u))du + |BD(t)K|\int_{-\eta(t)}^{0}(\tau(t+u)-\nu(t+u))du + \frac{1}{2}[\tau^2(t)-\nu^2(t)].\\ \{|D(t)KA| + |D(t)KB|\} + |D(t)K|\int_{-\tau(t)}^{-\nu(t)}\int_{s}^{0}(F(t+u)+\\ G(t+u))duds + \left|(D(t)K)^2\right|.\int_{-\tau(t)}^{-\nu(t)}\int_{s}^{0}(\tau(t+u)-\nu(t+u))du\,ds;\end{aligned} \tag{20}$$

2) *in the case of unstructured uncertainties,*

$$\begin{aligned}\mu(A+B+(\tau(t)-\nu(t))D(t)K) + \alpha(t) + \beta(t) + \eta(t)\{\|BA\| + \|B\|\,(\alpha(t)\\ +\beta(t)) + \|B^2\|\} + \|BD(t)K\|\int_{-\eta(t)}^{0}(\tau(t+u)-\nu(t+u))du + 0.5[\tau^2(t)\\ -\nu^2(t)]\{\|D(t)KA\| + \|D(t)KB\|\} + \|D(t)K\|\int_{-\tau(t)}^{-\nu(t)}\int_{s}^{0}(\alpha(t+u)\\ +\beta(t+u))duds + \left\|(D(t)K)^2\right\|.\int_{-\tau(t)}^{-\nu(t)}ds\int_{s}^{0}(\tau(t+u)-\nu(t+u))du < -\varepsilon.\end{aligned} \tag{21}$$

As before, if all parameters are supposed to be constant, the conditions are simpler:

Proposition 9 *The system (15) with constant parameters η, D, is robustly stabilized by (19) with constant τ, ν, if one of the following conditions holds for $t > t_0 + \xi$,*
1) *in the case of structured uncertainties with constant bounds,*

$$\begin{aligned}(A+B+(\tau-\nu)DK)^* + F + G + \eta\left\{|BA| + |B|\,(F+G) + |B^2| + |BDK|\,(\tau-\nu)\right\}\\ +\frac{1}{2}(\tau^2-\nu^2)\{|DKA| + |DK|\,(F+G) + |DKB|\} + \frac{1}{2}(\tau-\nu)(\tau^2-\nu^2)\left|(DK)^2\right| \text{ is Hurwitz;}\end{aligned} \tag{22}$$

2) *in the case of unstructured uncertainties with constant bounds,*

$$0 > \mu(A+B+(\tau-\nu)DK)+\alpha+\beta+\eta\{\|BA\|+\|B\|(\alpha+\beta)+\|B^2\|\}+$$
$$\|BDK\|\eta(\tau-\nu)+\frac{1}{2}[\tau^2-\nu^2]\{\|DKA\|+\|DKB\|+\|DK\|(\alpha+\beta)\}$$
$$+\frac{1}{2}(\tau-\nu)(\tau^2-\nu^2)\|(DK)^2\|. \quad (2$$

4 Conclusion

The problem of input-delayed, robust stabilization of systems with aftereffect has been studied in bo structured and unstructured cases. Original points are: 1) the possibility for the process or control involve distributed delays; 2) the consideration of possible uncertainties on the (constant or varyin delays; 3) the relative simplicity of the conditions in the time-invariant case. 4) the ability to stu multiple-delay case (even if we only studied here a special case (9) without discrete delay).

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